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The Extraction of Technical Coefficients from Input and Output Data

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ABSTRACT Presumably, input-output coefficients reflect technology, and these coefficients measure the input requirements per unit of product. This concept has been extended to consumption theory, where it models expenditure shares. Input-output coefficients are extracted from the national accounts of an economy, by taking average proportions between inputs and outputs. Since the latter represent all sorts of inefficiencies, this practice blurs the measurement of technology. Input requirements are better measured by minimal proportions between inputs and outputs. This approach separates the measurement of technology from that of productive efficiency.

KEY WORDS: Input-output coefficient

1. Introduction

What is an input–output coefficient? Presumably, it measures some input requirement per unit of some output. The inventor of input–output coefficients, Wassily Leontief, thought of them as recipes – to bake a pie you need so much flour, butter, sugar, etc. He thought the coefficients could be estimated by dividing the amount of consumed flour by the volume of produced pies and implemented this technique for the US economy. To date, this is still standard procedure. When applying thus constructed coefficients, the implicit assumption is that market shares between firms with different input–output proportions are constant. This assumption is not innocent. Not only may changes in the business climate favor more (or less) competitive firms, but the very concept of constant market shares is inconceivable in a world of multi-product firms with partially overlapping markets, particularly when the pattern of demand changes.¹

There are two issues. One is the problem that requirements are measured by average input/output proportions. In the author's opinion, a requirement better be measured by a minimal proportion. The other is that the multi-product nature of firms complicates

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the assignment of inputs to outputs and the reference set over which the average or the minimum is taken.

To address the choice between average and minimal input-output coefficients, consider a single-input/single-output industry. Firm i transforms l_i units of labor into x_i units of the product. Industry input and output are $\sum l_i$ and $\sum x_i$, respectively, and the standard way to determine the input-output coefficient is by division: $a = \sum l_i / \sum x_i$. This procedure amounts to taking a weighted average of the well-defined firm input-output coefficients, $a_i = l_i / x_i$. An econometric variant of the procedure is to estimate the ratio between inputs and outputs by regressing the inputs on the outputs without a constant term: $a = \sum l_i x_i / \sum x_i^2$.² It can be argued, certainly under the customary assumption of constant returns to scale, that the number of workers required per unit of product is determined by the firm that uses the least labor per unit of output: $a = \min l_i / x_i$. I believe that this statistic measures the production function. After all, a production function determines the maximum amount of output that can be produced with given input.

Suppose, without loss of generality, that firm 1 uses the least labor per unit of output. Then $a = a_1 = l_1 / x_1$. The other firms underperform. Take firm 2. It produces x_2 units of output and uses $l_2 = a_2 x_2$ units of input, but if it were to adopt the technique of firm 1, firm 2 would need only $l_2^* = a_1 x_2$ units of input. The inefficiency can also be measured on the output side. If we invert the input-output coefficients, we get output per unit of input, which is productivity. Firm 1 is the most productive firm: $1/a_1 = \max x_i / l_i$. Firm 2 uses l_2 units of input and produces $x_2 = (1/a_2)l_2$ units of output. If firm 2 were to adopt the technique of firm 1, it could produce $x_2^* = (1/a_1)x_2$ units of output, which is greater. Its output is x_2 but its potential output is x_2^* . The latter is the value of the production function. The slope of this function equals the productivity of firm 1 and the technical coefficient of firm 1 thus determines the production function.

We must bear in mind the aim of an input-output coefficient. One aim is to assess the employment impact of an increase in demand. Consider an increase in total demand from $\sum x_i$ to $\sum x_i + \Delta x$. The additional labor requirement is $a\Delta x$. What is the relevant input-output coefficient? The answer is: it depends. It is determined by which firm(s) pick(s) up the additional production. If all firms pick up a proportionate share, the standard input-output coefficient is appropriate. If the most competitive firm picks up the additional production, the technology based coefficient is relevant. The author finds that additional information is needed to answer this question. There is a range of answers. The technology-based coefficient sets the minimum employment impact. The maximum impact is attained if one employs the worst coefficient, of the least productive firm. To the author, the former seems more interesting. Ultimately it is a matter of projecting efficiency. This requires a forecast of the distance to the production possibility frontier and the determination of the latter must be based on technology coefficients.

The analysis becomes more complicated when firms produce different (combinations of) outputs. Consider a single input/double output industry with three firms, each employing one worker. Firms 1 and 2 are specialized, each producing one unit of each output, respectively. Firm 3 produces 0.75 unit of each product. What are the input-output coefficients for the two products? The standard procedure allocates the labor input of firm 3 equally to the two products, because their labor intensities are the same in view of the symmetry of the data. For each product the input is 1.5 and the output 1.75. The standard coefficients are $1.5/1.75 = 0.86$ for both products. Now let us replicate the input requirement and potential output calculations. Firm 1 produces one unit of

product 1. The only alternative to do so would be to adopt the technique of firm 3, but then it would require $4/3$ units of labor to produce the unit of product 1, which is more. Hence firm 1 is efficient. Firm 2 is efficient for the same reason. Firm 3 could adopt the techniques of firms 1 and 2 to produce its respective outputs, but dividing its labor in the specialized activities would yield 0.5 a unit of each product only, which is less than the 0.75 unit of each product it produces. Hence, firm 3 is also efficient! The input-output coefficients of firms 1 and 2 for products 1 and 2, respectively, are both 1, which is more than the standard value of 0.86. The reason is that firm 3 is more efficient than the combination of firms 1 and 2 in the production of the bundle of products 1 and 2. Its coefficients are $0.5/0.75 = 0.67$, which is less than the standard value.

Not only firm inefficiencies but also industrial organization inefficiencies blur the measurement of technology. Moreover, input–output coefficients differ by region in the input or output space. In the last example, the subset of firms 1 and 3 best produces both products if the demand for the first product is the greater. Then firm 1 determines the input-output coefficient of product 1, namely 1, and firm 3 uses $1 - 0.75$ units of labor for 0.75 units of product 2; the input-output coefficient of the latter is therefore $1/3$. If the demand for the second product is the greater, the Input–output coefficients become $1/3$ and 1. When the demands are equal, they are $2/3$.

The next section shows how the best practice input–output coefficients can be extracted from use and supply data conditional on the proportions of final demand. In the section after, it is shown how reductions in these coefficients measure technical change.

2. Inputs, Outputs, and the Production Function

The cumbersome variation of input-output coefficients with the proportions of the (net) outputs and the inputs of the economy leads the author to work with the production function instead of the input–output coefficients. There is a tight connection though. For example, if the input–output coefficients of some product are a_1, \dots, a_n , then the underlying production function is $\min(u_1/a_1, \dots, u_n/a_n)$, where u_1, \dots, u_n are the inputs. The production function measures potential output, which typically exceeds actual output.

\mathbf{U} and \mathbf{V} are the use and supply tables of dimension products by industries and have more rows than columns: they are long. However, since the author considers technical coefficients as representatives of best practices, it is essential not to aggregate firms' data into industries. The number of columns is not reduced to the number of industries, but remains the number of firms. The traditional use and supply tables – with columns aggregated to the level of industries – are not used in this analysis. The raw use and supply tables are also rectangular, but in the opposite direction, i.e. they are of the dimension ‘products’ by ‘firms’ and have more columns than rows: they are wide.³ The factor inputs are given by a table \mathbf{F} (dimension factors by firms).⁴

The vector of available factor inputs, \mathbf{f} , comprises the employed factor inputs, \mathbf{Fe} , plus possibly idle factor inputs. Here, the unit vector \mathbf{e} (of which all components are defined as 1) sums across firms. The net output is $\mathbf{y} = (\mathbf{V} - \mathbf{U})\mathbf{e}$. Given the factor inputs, how much more net output can be produced? In other words, how close is the economy to its frontier? This question will be answered keeping the proportions of final demand fixed. This is a conservative approach that steers the best practice coefficients proposed in this paper close to the conventional ones. The procedure can be given a theoretical foundation. Elsewhere (ten Raa, 2003), the author has shown that freeing Debreu’s (1951) coefficient of

resource utilization from individual household data requirements is equivalent to the imposition of Leontief preferences. Moreover, the rate of growth of the modified Debreu coefficient and the Solow residual are shown to add up to TFP growth.

Let potential output be a factor $1/\varepsilon$ greater. Here ε is the efficiency of the economy. For example, if potential output exceeds actual output by 25%, the economy produces only 80% of its potential. Let us reallocate the factor inputs, by running the firms at activity levels given by vector s (dimension number of firms). If the first component is 1.1, firm 1 operates at a 10% higher level. The actual activity vector is represented by $s = e$

$$\max_{s, \varepsilon \geq 0} e^T y / \varepsilon : (V - U)s \geq y / \varepsilon, \quad Fs \leq f \quad (1)$$

Net outputs are not valued at current prices. The inclusion of constant $e^T y$ (which is Gross Domestic Product, T is transposition) in the objective function is as inessential as the inclusion of any positive constant. It merely constitutes a monotonic transformation and proves useful for the price normalization. Indeed, the dual program associated with equation (1) is:

$$\min_{p^T, w^T \geq 0} w^T f : p^T (V - U) \leq w^T F, \quad p^T y = e^T y \quad (2)$$

The advantage of the dual program is its low dimension. The number of (price) variables is the number of inputs, or the number of products plus the number of factor inputs. The prices are competitive. For each firm, value-added is less than or equal to factor costs. In the first case, firms are shut down, in the second case firms are active.⁵ Denote the lower dimensional use and make tables of the active firms by U^* and V^* , respectively. Aggregate the factor input table, F , using the factor input prices, w , into row vector $w^T F$.

The aggregated linear program

$$\max_{s, \varepsilon \geq 0} e^T y / \varepsilon : (V - U)s \geq y / \varepsilon, \quad w^T Fs \leq w^T f \quad (3)$$

has the same solution as equation (1). In equation (3) the number of variables is the number of firms plus 1 for the expansion factor, while the number of constraints is the number of products plus 1 for the factor inputs. Consequently, there need be only as many active firms as there are products; in this case it follows that matrices

$$U^* \text{ and } V^* \text{ are square} \quad (4)$$

They represent the inputs and the outputs of the resource efficient providers of net output y . The commodity technology input-output coefficients are given by:

$$A^* = U^* V^{*-1}, \quad B^* = F^* V^{*-1} \quad (5)$$

On average, these coefficients are smaller than the standard input-output coefficients, based on U , F and V . Two warnings are in order. First, matrices A^* and B^* are contingent on final demand, y . Second, they imply neither material nor financial balance. For example, the implied requirements fall short of the observed ones. This casts some doubt on the balance axioms of Kop Jansen and ten Raa (1990).

3. Technical Change and Efficiency Change

Monitor the data through time. If the changes are small, linear analysis takes us back to input–output coefficients that are independent of the direction of the final demand changes. By the main theorem of linear programming, the primal and dual programs, equations (1) and (2), have equal values:

$$\mathbf{p}^\top \mathbf{y}/\varepsilon = \mathbf{w}^\top \mathbf{f} \quad (6)$$

Here, the price normalization constraint in equation (2) is used. Equation (6) is the macro-economic identity between the potential national product and income. Differentiating equation (6) with respect to time, using dots for time derivatives:

$$\dot{\mathbf{p}}^\top \mathbf{y}/\varepsilon + \mathbf{p}^\top \dot{\mathbf{y}}/\varepsilon - \mathbf{p}^\top \mathbf{y}\dot{\varepsilon}/\varepsilon^2 = \dot{\mathbf{w}}^\top \mathbf{f} + \mathbf{w}^\top \dot{\mathbf{f}} \quad (7)$$

Assuming that equation (1) is non-degenerate, the partition between the binding dual constraints, $\mathbf{p}^\top (\mathbf{V} - \mathbf{U})_{:j} = \mathbf{w}^\top \mathbf{F}_{:j}$ in equation (2), and the non-binding constraints (i.e. the other j columns), remains constant (ten Raa, 2005, section 4.6). Hence, $\dot{\mathbf{w}}^\top \mathbf{F}_{:j} = \dot{\mathbf{p}}^\top (\mathbf{V} - \mathbf{U})_{:j} + \mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})_{:j} - \mathbf{w}^\top \dot{\mathbf{F}}_{:j}$. Post-multiplication by the positive components of \mathbf{s} and inclusion of the zero components yields $\dot{\mathbf{w}}^\top \mathbf{Fs} = \dot{\mathbf{p}}^\top (\mathbf{V} - \mathbf{U})\mathbf{s} + \mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}$. In fact, by the phenomenon of complementary slackness and the constant partition, we may write $\dot{\mathbf{w}}^\top \mathbf{f} = \dot{\mathbf{p}}^\top \mathbf{y}/\varepsilon + \mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}$. Substitution into equation (7) and rearrangement of terms yields:

$$\mathbf{p}^\top \dot{\mathbf{y}}/\varepsilon - \mathbf{w}^\top \dot{\mathbf{f}} = \mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s} + \mathbf{p}^\top \mathbf{y}\dot{\varepsilon}/\varepsilon^2 \quad (8)$$

Division of equation (8) by equation (6) yields

$$\mathbf{p}^\top \dot{\mathbf{y}}/\mathbf{p}^\top \mathbf{y} - \mathbf{w}^\top \dot{\mathbf{f}}/\mathbf{w}^\top \mathbf{f} = [\mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}]/\mathbf{w}^\top \mathbf{f} + \dot{\varepsilon}/\varepsilon \quad (9)$$

On the left-hand side is Solow's residual expression for total factor productivity growth. The first term on the right-hand side is Domar's decomposition of technical change (Hulten, 1978). The last term is efficiency change. It is well known that productivity may grow as a result of technical change or efficiency improvements. The former effect represents the shift of technology and the latter a better allocation of the factor inputs. Only if we define input–output coefficients as best practice coefficients do we capture the efficiency effect.

The input–output coefficients were extracted by the active firms in equation (1), denoted by attachment of *'s to the input and output tables. Recall finding equation (4). Now substitute into the Domar term, the first term on the right-hand side of equation (9):

$$[\mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}]/\mathbf{w}^\top \mathbf{f} = [\mathbf{p}^\top (\dot{\mathbf{V}}^* - \dot{\mathbf{U}}^*)\mathbf{s}^* - \mathbf{w}^\top \dot{\mathbf{F}}^*\mathbf{s}^*]/\mathbf{w}^\top \mathbf{f} \quad (10)$$

Denote potential gross output by $\mathbf{x}^* = \mathbf{Vs} = \mathbf{V}^*\mathbf{s}^*$ and potential net output by $\mathbf{y}^* = \mathbf{y}/\varepsilon = (\mathbf{V} - \mathbf{U})\mathbf{s} = (\mathbf{V}^* - \mathbf{U}^*)\mathbf{s}^*$, where the first constraint in equation (1) is used.⁶ Now $\mathbf{p}^\top (\dot{\mathbf{V}}^* - \dot{\mathbf{U}}^*)\mathbf{s}^* = \mathbf{p}^\top [(\mathbf{V}^* - \mathbf{U}^*)\mathbf{s}^*] - \mathbf{p}^\top (\mathbf{V}^* - \mathbf{U}^*)\mathbf{s}^*$ and $\mathbf{w}^\top \dot{\mathbf{F}}^*\mathbf{s}^* = \mathbf{w}^\top (\mathbf{F}^*\mathbf{s}^*) - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}^*$. The two respective last terms are equal by the dual constraint in equation (2)

(which is binding for $s^* > 0$). Substitution in equation (10) yields:

$$\begin{aligned}
 & [\mathbf{p}^\top (\dot{\mathbf{V}} - \dot{\mathbf{U}})\mathbf{s} - \mathbf{w}^\top \dot{\mathbf{F}}\mathbf{s}] / \mathbf{w}^\top \mathbf{f} \\
 &= [\mathbf{p}^\top [(\mathbf{V}^* - \mathbf{U}^*)\mathbf{s}^*] - \mathbf{w}^\top (\mathbf{F}^*\mathbf{s}^*)] / \mathbf{w}^\top \mathbf{f} \\
 &= [\mathbf{p}^\top [(\mathbf{I} - \mathbf{A}^*)\mathbf{x}^*] - \mathbf{w}^\top (\mathbf{B}^*\mathbf{x}^*)] / \mathbf{w}^\top \mathbf{f} \\
 &= [(-\mathbf{p}^\top \dot{\mathbf{A}}^* - \mathbf{w}^\top \dot{\mathbf{B}}^*)\mathbf{x}^* + [\mathbf{p}^\top (\mathbf{I} - \mathbf{A}^*) - \mathbf{w}^\top \mathbf{B}^*]\dot{\mathbf{x}}^*] / \mathbf{w}^\top \mathbf{f}
 \end{aligned} \tag{11}$$

In the last term $\mathbf{p}^\top (\mathbf{I} - \mathbf{A}^*) - \mathbf{w}^\top \mathbf{B}^* = \mathbf{p}^\top (\mathbf{I} - \mathbf{U}^*\mathbf{V}^{*-1}) - \mathbf{w}^\top \mathbf{F}^*\mathbf{V}^{*-1} = \mathbf{p}^\top (\mathbf{V}^* - \mathbf{U}^* - \mathbf{w}^\top \mathbf{F}^*)\mathbf{V}^{*-1} = 0$, again by the dual constraint in equation (2) (binding for $s^* > 0$). Consequently, substitution of the Domar term (equation (11)) in equation (9) yields:

$$\mathbf{p}^\top \dot{\mathbf{y}} / \mathbf{p}^\top \mathbf{y} - \mathbf{w}^\top \dot{\mathbf{f}} / \mathbf{w}^\top \mathbf{f} = -(\mathbf{p}^\top \dot{\mathbf{A}}^* + \mathbf{w}^\top \dot{\mathbf{B}}^*)\mathbf{x}^* / \mathbf{w}^\top \mathbf{f} + \varepsilon / \varepsilon \tag{12}$$

This is Wolff's (1985) technical coefficient reductions representation of productivity growth, but augmented with efficiency change.

4. Conclusion

Technical coefficients control the input requirements of the outputs or the output that is producible with the inputs, and therefore they need to be determined by the best-practice firms. A linear program that determines how much output can be produced given the intermediate and factor inputs identifies the best-practice firms. The final-output-factor-input ratio of the economy, i.e. productivity, can be increased by reductions of these technical coefficients or by efficiency improvements.

A contribution of the musings in this paper is the separation of the dual roles of input coefficients, describing technology and reflecting cost structures. The former role is best played by best-practice coefficients, the latter by average proportions. There are critical differences. Standard input-output table coefficients add to unity, even at constant prices if the double deflation methodology is applied, but best practice coefficients do not. Much of the fun is in the differences. In theory, they catch the inefficiencies.

The proof of the pudding is in the eating. I hope a reader will attempt to implement this proposal. There is also more theory to be done. A main complication foreshadowed here is the role of balancing. Best practice coefficients are, but should not be, balanced. However, it will be difficult to disentangle errors of measurement from true cost advantages or disadvantages. Stochastic frontier estimation has the potential to overcome this complication and it is hoped another reader will work it out.

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Notes

¹See ten Raa and Rueda-Cantuche (2007a) for a recent review of the construction of input-output coefficients.

²Mattey and ten Raa (1997) and ten Raa and Rueda-Cantuche (2007b) construct confidence intervals for the coefficients and their Leontief inverse. In principle, this can also be done for the standard formula.

³A referee notes that in Sweden the number of enterprises is close to 1 million and that the number of products is indeed less. Products by enterprise supply and use tables are not, and should not, balance.

Valuations should be in basic prices. ten Raa and Rueda-Cantuche (2007a) detail the procedure, including the assumed equality of margins and net commodity taxes between establishments in a given industry, consuming a given commodity.

⁴Normally, capital input is calculated by first using, for example, the perpetual inventory method to get the productive stock by asset type and then estimating the respective user costs in one way or other. This requires long investment series that are hardly available for single enterprises and may require industry proxies.

⁵This is the phenomenon of complementary slackness. For the theory of linear programming see ten Raa (2005, Chapter 4).

⁶If it is non-binding, the slack will be killed by the shadow prices in the Domar effect.

References

- Debreu, G. (1951) The coefficient of resource utilization, *Econometrica*, 19, pp. 273–292.
- Hulten, C.R. (1979) Growth accounting with intermediate inputs, *Review of Economic Studies*, 45, pp. 511–518.
- Kop Jansen, P. and ten Raa, Th. (1990) The choice of model in the construction of input–output coefficients matrices, *International Economic Review*, 31, pp. 213–227.
- Mattey, J. and ten Raa, Th. (1997) Primary versus secondary production techniques in U.S. manufacturing, *Review of Income and Wealth*, 43, pp. 449–464.
- ten Raa, Th. (2003) Debreu's coefficient of resource utilization, the Solow residual, and TFP: the connection by Leontief preferences, Center Discussion Paper 111.
- ten Raa, Th. (2005) *The Economics of Input-Output Analysis* (Cambridge: Cambridge University Press).
- ten Raa, Th. and Rueda-Cantuche, J. (2007a) A generalized expression for the commodity and the industry technology models in input-output analysis, *Economic Systems Research*, 19, pp. 99–104.
- ten Raa, Th. and Rueda-Cantuche, J. (2007b) Stochastic analysis of input-output multipliers on the basis of use and make matrices, *Review of Income and Wealth*, 53, pp. 318–334.
- Wolff, E. (1985) Industrial composition, interindustry effects, and the U.S. productivity slowdown, *Review of Economics and Statistics*, 67, pp. 268–277.