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Emissions and footprints:
Resolving the confusion using national accounts

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Abstract

I resolve the confusion surrounding emissions and footprints. Footprints impute industry emissions to end users (households). Product footprints are shown to be equal to emission intensities inflated by the Leontief inverse. However, unlike the prevailing practice of employing industry-by-industry dimensions, the input-output coefficients must have product-by-product dimensions and be constructed according to the product technology model. The analysis shows that an emission tax increases the product prices proportionally to the product footprints.

Keywords: Footprint analysis, emission taxes

1. Introduction

The concepts of emissions and footprints are confused. Products are promoted with the slogan of 'zero emissions.' However, their footprints may be high, think of the battery of an electric vehicle. So, which is the relevant concept and, what is their relationship? Roughly speaking, emissions have the dimension of emittants (e.g., CO_2, measured in tons) by production (e.g., power plants) and footprints have the dimension of emissions (CO_2 tons) implied by consumption (e.g., household expenditures on products). Emissions are to be understood as reductions of the stock of a natural resource, e.g., clean air or water. Then footprints locate the sources of these reductions and the concept is widely applicable, including to water footprints and sustainability analysis. The main source of the confusion is the neglect of the indirect production effects of the claims of products on natural resources.

This dimensional difference is similar to the accounting distinction between the national income and the national product. Hey, aren't they equal? Well, they are in the aggregate, as we learn in macroeconomics, but the dimensions differ. Roughly speaking, the dimension of national income is income earned in production (value-added), while the dimension of national product is expenditures by consumers (final demand). The respective totals are equal indeed, as I will explicate further down in this paper. However, respective elements differ, even when we classify producers by their respective products and use the same product classification for final demand. The most prominent case is that of business services. Income earned in business services is high, but its contribution to the national product is zero, because final services demand, exercised by consumers, is for personal services.

The national product measures only final demand, also called net output. The production of services is absorbed by producers and consumers. The first part is intermediate demand, and the second part is final demand. The same distinction between intermediate and final demands plagues the related yet different concepts of emissions and footprints. An emission intensity is, say, CO_2 tons per million dollars of gross output, be it for intermediate demand (industries) or final demand (households). A footprint intensity is CO_2 tons per million dollars million dollars of net output. Because net output of an economy is less than gross output, footprint intensities are greater than emission intensities, by a factor equal to the gross output/net output ratio. This ratio is called the Domar ratio [1] and plays a similar role in the aggregation of productivities across industries. However, if emissions and footprints are measured not in intensities but in levels (CO_2 tons) and aggregated across industries households, respectively, one obtains the same total. In short, footprints impute industry emissions to end users.

The imputation of emissions to consumption products is done by input-output analysis, a quantitative technique developed by Wassily Leontief [2], who applied it to environmental economics [3]. It still serves as a useful introduction, in the next section. The standard input-output model implicitly assumes that each industry produces a single output. Only recently, ten Raa and Stahlie [4] have shown how to handle modern national accounts, which feature industries with overlapping products. This will be exposited in section 3.

2. Emissions and simple footprint analysis

A traditional environmentally extended input-output table is as follows.

	Industry 1	…	**Industry *n***	**Final demand**	**Total**
Product 1	x_{11}	…	x_{1n}	y_1	x_1
. . .	. . .		. . .	. . .	. . .
Product *n*	x_{n1}	…	x_{nn}	y_n	x_n
Value added	v_1	…	v_n		
Total	x_1	…	x_n		
Emissions	d_1	…	d_n		

Table 1. Traditional input-output table

Let me explain how Table 1 is built. All entries are in million dollars, except the emissions. The first row shows the sales of the product of industry 1 to the n industries. The total output is entered in the last column. The next to last column is the residual output, that accrues to final demand. Now look at the first column. The first n entries are industry 1's expenditures on product inputs. The total output of industry 1 is copied in the next to last entry. The residual between sales and product expenditures is entered in the value-added row. Last, but not least in environmental modeling, the industry emissions are in tons and entered in the bottom row.

The national product is the total of the final demand column. The national income is the total of the value-added row. Both are equal to the grand total, $x_1 + \ldots + x_n$, minus the total of the square sub-table of intermediate deliveries, $(x_{11} + \ldots + x_{1n}) + \ldots (x_{n1} + \ldots + x_{nn})$. Hence the national product and the national income are equal, but only as aggregates.

Input-output analysis is simple. Assume constant returns to scale, then the inputs of industry 1 are proportional to its output, with input coefficients $a_{11}, \ldots, a_{n1}$. This is the first column of the input-output matrix A; and likewise, the other columns. By the same constant returns to scale assumption industry demand for product 1 is $a_{11}x_1 + \ldots + a_{1n}x_n$. Organize total output in vector x and final demand in vector y. Then industry demand is Ax and, therefore,

$$x = Ax + y \tag{1}$$

Equation (1) relates gross output x and net output y. Organize the industry emissions in row vector d. By the assumption of constant returns to scale, the emission of industry 1 is proportional to its output, with emission coefficient s_1. This is the first entry of the emission intensity row vector s. Denote the summation vector by e (all entries equal to 1). Then total emission equals

$$de = sx \tag{2}$$

Equation (2) shows the emissions by industries, $s_1x_1 + \ldots + s_nx_n$. The transformation to footprints runs as follows. The solution to equation (1) is $x = By$, where $B = (I - A)^{-1}$, the so-called Leontief inverse of A. In fact, if the residual y or the residual v is positive, then

$$B = (I - A)^{-1} = I + A + A^2 + \ldots \tag{3}$$

It follows that total emission equals

$$de = sBy = fy \tag{4}$$

In equation (4), f is the row vector of product footprints, defined by $f = sB$. By equation (3) we obtain $f = s + sA + sA^2 + \dots$. Thus, the row vector of product footprints equals the row vector of emission coefficients, plus the emission contents of the inputs, plus the emission contents of the inputs of the inputs, etcetera. The product footprints, the f components, are greater than the first term, the emission coefficients, the respective emission coefficients of s. Oftentimes the indirect effects are neglected [5].

3. Emissions and modern footprint analysis

Leontief's input-output table was the basis of national accounting. However, the identification of industry outputs with consumption products was a pain in the neck for national accountants. Modern firms not only have multiple inputs, but also multiple outputs. So, the question was, how classify firms in industries? The simple but brilliant idea of Richard Stone was to distinguish the dimensions of products and activities [6]. Instead of Leontief's tightly organized input-output table statistical offices nowadays make separate input and output tables.

The System of National Accounts features an input table U (U for use) and an output table V (the next letter in the alphabet) [7]. The dimension of each table is nowadays product-by-industry. (The output table used to be industry-by-product [6].) The framework is flexible, allowing for industries producing each other's products. In principle the framework may even allow for different numbers of products and of industries, but I relegate that further extension to remarks after the analysis. So let there be n industries producing n products. The first column of the input table U shows the inputs of industry 1 and the first column of the output table shows the outputs of industry 1, etcetera.

The total output vector, x, is a product vector, obtained by aggregating the output table with respect to the industries: $x = Ve$. The input vector is also a vector, obtained by aggregating the input table with respect to the industries: Ue. The final demand vector is the residual between the two, i.e.,

$$y = (V - U)e \tag{5}$$

The matrix on the right-hand side of equation (5) is called the net output table of an economy. It is the core of the national accounts of an economy. The dimension of the net output table is product-by-industry. The post-multiplication of the net output table by summation vector e aggregates over industries and yields national product vector y. Similarly, pre-multiplication of the net output table by summation row vector e' (' for transposition) would yield the value added by industry

$$v = e'(V - U) \tag{6}$$

It follows from equations (5) and (6) that

$$e'y = ve \tag{7}$$

Equation (7) is the national product-national income identity.

As before, the industry emissions are in row vector d. Using equation (5), total emission equals

$$de = d(V - U)^{-1}(V - U)e = d[(I - UV^{-1})V]^{-1}y = dV^{-1}(I - UV^{-1})^{-1}y \tag{8}$$

Now define the emission intensity row vector s and the input-output matrix A as follows:

$$s = dV^{-1},\ A = UV^{-1} \tag{9}$$

There are many ways to construct emission intensities and input-output coefficients, and equation (9) is only one of them. However, Kop Jansen and ten Raa have shown that construct (9), the so-called product technology model, is implied by sets of balance and invariance axioms [8].

Equation (9) generalizes the emission intensity row vector and the input-output matrix we introduced in section 2, where industries were single output producers. Substituting equations (9) and (3), and the row vector of product footprints $f = sB$. we see that equation (8) reduces to equation (4).

So, equation (4) remains useful for modern footprint analysis. Mind the dimensions though. Since d is an industry row vector and V is product-by-industry, s is a product row vector. Since U and V are product-by-industry, A, and hence B, have the dimension of product-by-product. While the emissions, d, are by industry, product footprints, $f = sB$, are by product, as should be. However, applied footprint analysts use input-output matrices with the wrong dimension, namely industry-by-industry [9]. Some applied footprint analysts do use product-by-product tables but apply a different formula than equation (9), which yields a bias [4].

Recent analysis of the Dutch economy applies the row vector of the $n = 80$ product footprints to the 80-dimensional budget share vectors of the ten income classes (deciles), see Table 2 [4, 10].

Table 2: CO_2 footprints per euro expenditure of the 10 Dutch income classes

Income classes (deciles)	Household income (m€)	Footprint (ton/m€)
1	0.0095	325.45
2	0.0172	321.83
3	0.0206	321.44
4	0.0236	315.52
5	0.0270	316.50
6	0.0304	322.94
7	0.0342	320.17
8	0.0386	319.14
9	0.0450	317.54
10	0.0754	303.24
All private households	0.0321	316.67

The last column of Table 2 shows that the footprint per euro expenditure decreases with income, as will be detailed now. Regressing the expenditure-based footprint on income the constant term is $\alpha = 327.62$ and the regression coefficient is $\beta = -287.5$, where β is significantly negative with a (one-sided) p-value of only 0.001. Coefficient β measures the reduction of (expenditure-based) footprint in ton/m€ when income increases by a unit, i.e., 1 m€, or, considering an income raise of a ten thousand euros, the expenditure-based footprint decreases on average by 2.875 ton/m€ (i.e., 0.02875 ton/ten thousand€).

What if the input and output tables are not square? Obviously, there are two possibilities.

(i) There are more activities than products.
(ii) There are less activities than products.

Footprint analysis essentially imputes activity emissions to consumer products; there are as many footprints as there are products. As we have seen, if the number of activities equals the number of products, the imputation can be done, see equations (4) and (8). The number of equations matches the number of unknowns. In case (ii), however, we are short of equations and cannot impute footprints to the many products. And in case (i) we have more equations than unknowns. This means that it will not be possible to get a perfect model fit. Errors will emerge. The good news is that the errors accommodate econometric analysis, with the advantage that confidence intervals can be given for footprints.

Published input and output tables, both economic and environmental, are of type (ii); activities are industries. The "solution" to the problem of under-identification is to reduce the number of products to the number of industries, a practice that yields an aggregation bias. Luckily, the raw data used by statistical offices in the construction of published tables are of type (i). Activities are firms or even business units. So, it would be extremely useful to provide access to the raw data, to facilitate econometric analysis. Mattey and ten Raa were thus able to estimate input-output coefficients, but only because Joe Mattey's status as Federal Reserve employee gave us access to Manufacturing Census data [11]. Things are opening up. A successful environmental-econometric study is Rueda-Cantuche and Amores [12].

4. Environmental policy

There are many environmental policies. Subsidies for clean technologies, allocations of emission caps, emission taxes, and emission rights trades. Subsidies are inefficient [13]. Allocations of emission caps are surrounded by lengthy political negotiations. Pigouvian emission taxes are efficient and emission rights trades can be shown to be equivalent [14, 15]. So let me focus on Pigouvian taxes.

The taxes will trickle down to the consumers prices and reduce purchasing power. The analysis runs as follows. Thus far the economic accounts are in nominal values, i.e., in current prices: all product prices are equal to one. Now introduce a tax t per unit of emission. Industry costs were on the product inputs (the column totals of the input table) and the factor inputs (value added). The price equation was equation (6). Here the price row vector was e'. The tax will increase the price row vector to, say, p. Price equation (6) turns

$$v + td = p(V - U) \tag{10}$$

Equation (10) shows that the prices change such that value added, $p(V - U)$, will cover not only the factor costs, v, but also the emission tax, td. (Recall d is the row vector of industry emissions and t is the rate at which they are taxed.) Subtract equation (6) from equation (10):

$$td = (p - e')(V - U) \tag{11}$$

Invert the net output table in equation (11):

$$p - e' = td(V - U)^{-1} = td[(I - UV^{-1})V]^{-1} = tdV^{-1}(I - UV^{-1})^{-1} \tag{12}$$

Substitute formulas (9) and (3) in equation (12):

$$p - e' = tsB = tf \tag{13}$$

Equation (13) shows that the price changes due to an emission tax are proportional to the respective product footprints, $f = sB$ (s the emission intensities and B the Leontief inverse), and that the proportionality coefficient is equal to the emission tax rate, t, for all products.

While the calculation of footprints is cumbersome, involving a matrix of input-output coefficients and Leontief inversion, these computations are not required to implement footprint-based taxes. The market can do so. The mechanism is the same as the collection of a value-added tax. The only difference between a value-added tax and an emission tax is that the value-added coefficients are replaced by emission coefficients. In both cases the tax is passed on to downstream industries and thus inflated by the Leontief inverse. However, there is no need to calculate the Leontief inverse. Firms receive a tax on each unit of output, subtract the taxes paid on the intermediate inputs, and the taxman collects the difference. This mechanism requires no calculation. A great advantage of the value-added tax is that the deductibility of taxes paid to supplying firms offers a wealth of information to the tax authority. Likewise, emission taxes paid by firms to suppliers better be deductible as well.

The principle behind the Pigouvian tax is "the polluter pays," but the tax is passed on to the consumers who will face price increases proportional to the footprints. The tax will be regressive if and only if footprints per dollar expenditure decrease with income, which is the case for the Dutch economy, see Table 2, last column, and the subsequent regression analysis.

5. Conclusion

I have resolved the confusion surrounding emissions and footprints. Footprints impute industry emissions to end users (households). Emissions are proportional to the gross outputs of industries, while footprints are proportional to the net outputs consumed by households. Since the net outputs are smaller than the gross outputs, the product footprints are greater than the emission intensities. In fact, the product footprints equal the emission intensities inflated by the Leontief inverse. Unlike the prevailing practice of employing industry-by-industry dimensions, the input-output coefficients must have product-by-product dimensions and be constructed according to the product technology model. The analysis shows that an emission tax increases the product prices proportionally to the product footprints.

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