SYNERGETIC AND RESONANCE ASPECTS OF INTERDISCIPLINARY RESEARCH

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"Possibly already stars and the cores of galaxies, but certainly ecosystems, the world-wide Gaia System of the bio-plus atmosphere ..., social systems, civilisations and cultures are no less dissipative self-organizing systems than are ideas, paradigms, the whole system of science, religions and the images we hold of ourselves and of our roles in the evolution of the universe ... ."

Erich Jantsch in M. Zeleny, Autopoiesis, Dissipative Structures and Spontaneous Social Orders, Westview Press, Boulder, 1980, p. 86

In this essay we shall entertain the thought that the whole system of science has a certain dynamics. The dynamics is made of developments of disciplines intertwined with cross effects, that is, interdisciplinarity. Science does not, however, march down a unique path of development; from certain points on, disciplines may nurture or frustrate each other. Therefore, interdisciplinary research itself may be subjected to bifurcation analysis!

History of science students note that at certain points disciplines merge. For example, at the turn of the century political economy began to employ mathematics and, in return, to breed new chapters of mathematics such as programming, game theory and even some parts of matrix algebra. Two disciplines, traditionally distinct, hooked up; their developments are now

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simultaneous and may be integrated to some extent. Interdisciplinarity is the buzz word for all this.

The use of mathematics clarifies economic issues and helps in solving problems of economic theory and policy. Therefore, mathematics may speed up the development of economics, but excessive use of mathematics by economists can have choking effects. The subsequent analysis illustrates this.

Consider two disciplines of science; "indicators", say m(t) and e(t), which may vary from time to time (t), are introduced as follows. Each discipline accumulates knowledge; common parlor for the stock of knowledge is "the state of the art". The growth rate of the state of the art is, of course, a (relative) flow variable; it stands for the accumulation of knowledge in the discipline and is denoted m(t) or e(t). So these symbols do not refer to the states of both arts or the sizes of libraries, but to the process of accumulation or research; m(t) is the research indicator for one discipline, e(t) for the other.

It is usually assumed that the creation of knowledge is an irreversible process. Once an invention has been made, it needs not be redone. Knowledge does not wear of tear; its accumulation rates, m(t) and e(t), are nonnegative. Yet research goes up and down; its indicators fluctuate. Which forces govern these fluctuations? In other words, what are the equations for $\mathring{m}(t)$ and $\mathring{e}(t)$, where $^{\circ}$ denotes time differentiation.

It will be fruitless to aim at a single law of research which holds forever; the "constants" of research, and interdisciplinary research in particular, adjust from time to time. Instances of such adjustments are the so-called scientific revolutions: a major new paradigm affects the development of science. But in between such revolutions research has a dynamics which may be characterized by means of constants; this dynamics is mathematically described by an autonomous differential equations system with parameters: various stages of science correspond to different values of the parameters; this, at least, is one way to model stages of science.

We shall now begin to construct the equations. For focus we will consider variations of only interdisciplinary parameters; the internal disciplinary parameters are fixed numerically. Specifically, we take

$$\mathring{m} = m(1-m), \tag{1}$$

$$\mathring{e} = -e. \tag{2}$$

For either discipline, zero research yields no development. Otherwise we may divide through by m and e, respectively, to obtain logarithmic time derivatives and to note the following. The state of the art in the 'm'-discipline has a natural rate of growth equal to one; this is the value of the resonance parameter which, by definition, measures the internal rate of development. If the natural rate is overshot, then 1-m is negative and the discipline slows down; if research is relatively suppressed (m less than one), then 1-m is positive and there is a tendency to catch up. The 'e'-discipline, however, has a hopeless internal dynamics; this discipline, left by its own, contracts its research, until it has exhausted merely a limited amount of the unknown. The value of the resonance parameter or the natural rate of growth of the state of the 'e'-art is zero.

So far we have considered the disciplines in separation, as if they were in the old stage; but now consider the emergence of interdisciplinarity. It seems natural to represent it by the product of the two research indicators, me, the most simple expression in which the indicators reinforce each other. However, the impact on the two disciplines may differ; let it be μme and εme, respectively: μme is inserted in the right hand side of the 'm'-equation, εme is the right hand side of the 'e'-equation. μ and ε are interdisciplinary propensities or synergetic parameters (Greek for "working together"), which, again by definition, measure the interdisciplinarity impact on a unit base.

Although the synergetic parameters will be different, they may be related to each other. Here we have in mind the reluctance of many mathematicians to cooperate with eager economists, out of fear that excessive interdisciplinarity is decremental for mathematical research. Hence μ may be negatively correlated with ε, like $\mu = 1 - \varepsilon$. In sum, we postulate the following laws of research:

$$\dot{m} = m(1-m) + (1-\varepsilon)me, \tag{3}$$

$$\dot{e} = -e + \varepsilon me. \tag{4}$$

The laws can be simplified by introducing the logarithmic rate of change operator, $^{o} = \frac{d\log}{dt}$. Then we obtain the Lotka-Volterra equations

$$\overset{o}{m} = 1 - m + (1-\varepsilon)e, \tag{5}$$

$$\overset{o}{e} = \varepsilon m - 1. \tag{6}$$

We will distinguish various cases. $\varepsilon=0$ refers to the old stage of science in which the e-discipline is distinct from the m-discipline though the latter receives impulses from the former. When ε becomes positive we enter the stage of science which is marked by a moderate degree of interdisciplinarity: $0<\varepsilon<1$. $\varepsilon=1$ represents a scientific revolution, as we shall see below. Then $\varepsilon>1$ is entered, the stage marked by an "excessive" flurry of interdisciplinary activity on the side of the 'e'-discipline. The first case, $\varepsilon=0$, is similar to the case that has been discussed already. The other cases are represented by phase diagrams in Figures 1, 2 and 3.

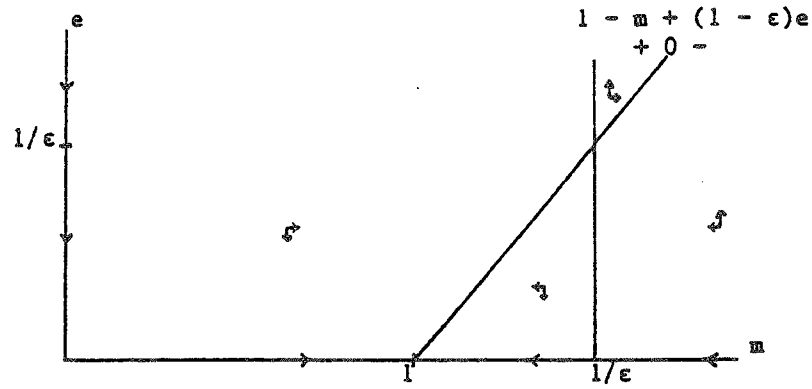

Figure 1. Moderate interdisciplinarity: $0<\varepsilon<1$.

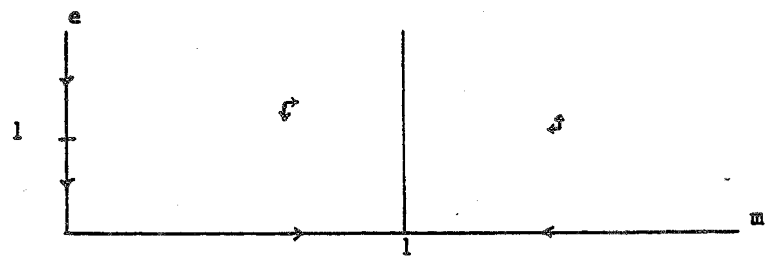

Figure 2. Scientific revolution: $\varepsilon=1$.

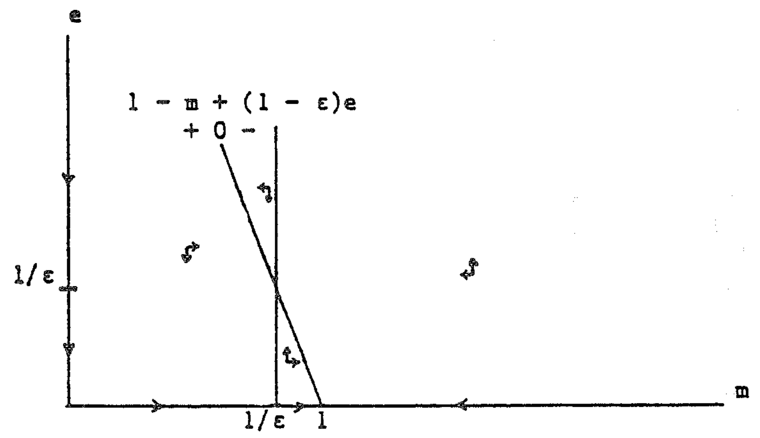

Figure 3. Excessive interdisciplinarity: ε>1.

Throughout, there are three stationary solutions for (m,e), namely (0,0), (1,0) and (1/ε,1/ε). In the stage of moderate interdisciplinarity (Figure 1) only (1,0) is stable. In the stage of excessive interdisciplinarity (Figure 3) only (1/ε,1/ε) is stable. All these points are plotted in Figures 4 and 5 as functions of ε. The figures also indicate the (in)stability of the points.

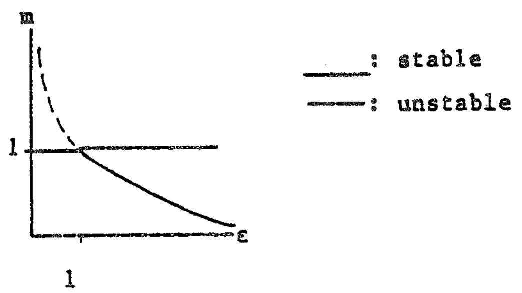

Figure 4.
Stationary m-solutions

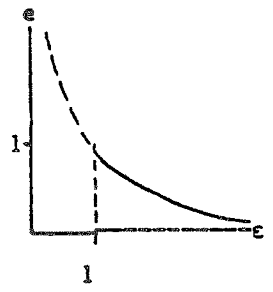

Figure 5.
Stationary e-solutions

We shall briefly discuss the nature of stable stationary points. For $0<\varepsilon<1$, $(m,e) = (1,0)$ is an attractor; Figure 1 shows that this is true at least for the region where m and e are both less than $1/\varepsilon$, so the system of science will tend to the stationary state $(m,e) = (1,0)$. Next consider $\varepsilon>1$. Figure 3 hints that $(1/\varepsilon,1/\varepsilon)$ is the center of counter-clockwise orbits. In fact, there is a Hopf bifurcation which amounts to a family of orbits which by the Poincaré-Bendixson theorem spiral down to $(1/\varepsilon,1/\varepsilon)$ or possibly a limit cycle about it[2]. Starting at a lower side, interdisciplinarity gives a big push to the 'e'-discipline; however, this has a slightly decremental effect on the 'm'-discipline which consequently slows down a bit. This, on its turn, dries up the source of new results for the 'e'-discipline which, lacking an own positive force, tumbles down; the 'm'-discipline is now freed from competitive pressures and progresses on its own force towards its natural rate of research. Then the cycle is repeated.

We have seen in Figures 4 and 5 that at $\varepsilon=1$ two solution families bifurcate from $(m,e) = (1,1)$. However, both for $\varepsilon<1$ and for $\varepsilon>1$ only one branch was stable; in the latter case this was $(1/\varepsilon,1/\varepsilon)$. We have also seen that this branch was accompanied by a family of spirals, which is essentially a further bifurcation. It is a natural question to ask what occurs to science when, departing from the stage of moderate interdisciplinarity $(0<\varepsilon<1)$ and assuming (m,e) has been attracted already towards $(1,0)$, the synergetic parameter, ε, is increased to an "excessive" value greater than one. In the transition there is the scientific revolution $(\varepsilon=1)$ which is completely unstable as Figure 2 reveals and even structurally unstable since infinitesimal parametrical change brings about a different picture, like Figure 1 or 3. Then science may be kicked around anywhere. Therefore we do not know which orbit will be traced out when the excessive value of the synergetic parameter is arrived at. But eventually the system will settle down to a state of either constant or periodic research as Hirsch and Smale conclude for a similar case.

Finally it should be said that, especially in the field studied, bifurcations are apt to produce themselves in different variations. Two of them have been revealed and discussed in this essay: multiple (stable) singular parts and structural instability of the system; the latter would be increased if the system were to be disturbed by extra non-linearities (apart from the bilinearity already present), or by hypothesising that the coefficients could be functions of the state variable. A third possibility is that, instead of the system of differential equations (5)-(6) one thinks of a system of differential correspondences, more then one possible timepath originating from each point (or some points) in the phase plane, but this possibility is left for further investigation.

Anyway, we have classified various pathes of disciplinary developments which bifurcate in our model of science dynamics, so we have some idea of what the possibilities are. It should be mentioned that this is contingent on our specification of the laws of research; this specification is, of course, just an image we hold of ourselves. And how this image will evolve, bifurcate, ...?

NOTES AND REFERENCES

1) The authors are grateful to Sebastian van Strien for valuable discussions. Netherlands Organization for the Advancement of Pure Research (Z.W.O.) support to the second author is gratefully acknowledged

2) For this we refer to M.W. Hirsch and S. Smale: (1974), Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, New York, pp. 264-265.