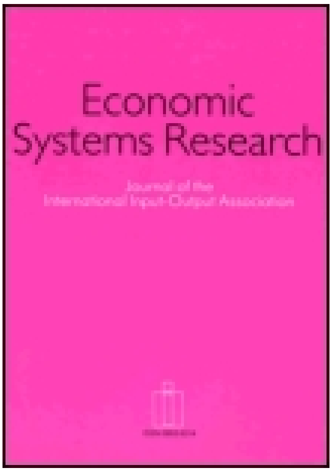

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COMPLEMENTARITY IN INPUT–OUTPUT ANALYSIS AND STOCHASTICS

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The complementarity between the quantity and value systems of input–output analysis is shown to be the basis of the complementarity problem approach to computable general equilibrium. The numerical superiority of the latter to the linear programming approach facilitates stochastic analysis of input–output scenarios. For the example where Kyoto targets are underachieved to uncertain degrees, confidence intervals are derived for the associated consumption reductions.

Keywords: Complementarity problem; Stochastic input–output analysis

1. INTRODUCTION

ten Raa and Shestalova (2014) use the complementarity problem to determine the equilibrium prices and quantities for mutually trading economies. The purpose of this sequel paper is twofold. First, we provide an input–output foundation to this approach. The complementarity between the quantity and value systems of input–output analysis naturally suggests itself as a basis of the complementarity problem approach to computable general equilibrium and we show that this works out indeed. Second, we demonstrate that the complementarity problem approach makes possible stochastic input–output analysis. In standard models, equilibrium is determined using a fixed point algorithm, either in commodity space or in utility space. In the first case, prices are adjusted as to equate commodity supply and demand. The second case exploits the welfare theorems (by which equilibria are efficient). It constructs a social welfare function, determines the efficient allocation and the supporting prices, compares the values of the consumption bundles to the values of the endowments, and adjusts the weights in the social welfare function as to equate the two values for each type of consumer. The welfare approach is more indirect, but numerically more efficient, because the dimension is lower; for example, in international trade models, the dimension is the number of countries or regions, which is smaller than the number of commodities – the 28 documented Chinese provinces with 50 industry input–output accounts each could thus be analysed by ten Raa and Pan (2005), in particular the effects of competitive markets on income inequality. The nonlinear complementarity problem routine of GAMS (Ferris and Munson, 2000) further reduces the time to solve

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big economic models, from minutes to milliseconds, and this observation suggests Monte Carlo analysis of big input–output systems. The approach is useful for applied modellers, whose analyses and reports could benefit from the additional insights concerning the sensitivity of the modelling outcomes to the measurement of emissions and environmental damages.

2. COMPLEMENTARITY IN INPUT–OUTPUT

The input–output model rests on the duality between prices and quantities and has been criticized for analysing the two independently. In a recent book, however, Bródy (2005) uses the duality to connect prices and quantities in a symmetric way. We apply his idea that excess supply reduces price and excess cost reduces output, not by setting up differential equations, but by introducing complementarity.

Allowing for slack, the quantity system is $(\mathbf{I}-\mathbf{A})\mathbf{x} \geq \mathbf{y}$, where $\mathbf{x} \geq \mathbf{0}$ and $\mathbf{y}$ are the gross and net output commodity vectors (n-dimensional) and $\mathbf{A}$ is the input–output coefficients matrix, and the value system is $\mathbf{p}(\mathbf{I}-\mathbf{A}) \leq \mathbf{v}$, where $\mathbf{p} \geq \mathbf{0}$ and $\mathbf{v}$ are the price and factor cost row vectors. If a commodity is in excess supply (strict inequality in a component of the quantity system), the price of that commodity will be zero: $[(\mathbf{I}-\mathbf{A})\mathbf{x}]_i > y_i \Rightarrow p_i = 0 \;\; (i = 1, \ldots, n)$, that is, *complementarity* between the slacks in the supply and the price inequalities. This is equivalent to $p_i[(\mathbf{I}-\mathbf{A})\mathbf{x}-\mathbf{y}]_i = 0 \;\; (i = 1, \ldots, n)$ and, because a sum of nonnegative terms is zero if and only if each term is nonzero, to the single equation $\mathbf{p}[(\mathbf{I}-\mathbf{A})\mathbf{x}-\mathbf{y}] = 0$, that is, *orthogonality* of price and excess supply. The complementarity between the quantity system and prices is succinctly written as $\mathbf{0} \leq \mathbf{p} \perp (\mathbf{I}-\mathbf{A})\mathbf{x}-\mathbf{y} \geq \mathbf{0}$.

Similarly, if a commodity incurs a loss (strict inequality in a component of the value system), the supply of that commodity will be zero, that is, complementarity between the slacks in the loss and output inequalities, $\mathbf{0} \leq \mathbf{x} \perp \mathbf{v}-\mathbf{p}(\mathbf{I}-\mathbf{A}) \geq 0$. The pair of orthogonality conditions implies $\mathbf{py} = \mathbf{vx}$, which is the identity between national product and national income. If prices $\mathbf{p}$ and quantities $\mathbf{x}$ are positive, the material and financial balances hold, $(\mathbf{I}-\mathbf{A})\mathbf{x} = \mathbf{y}$ and $\mathbf{p}(\mathbf{I}-\mathbf{A}) = \mathbf{v}$. Many input–output economists analyse these two equations in isolation – to derive gross output from net output and to derive price from factor cost – but in equilibrium analysis, the price and quantity systems interact.

In the commodity market, the price and quantity variables are $\mathbf{p}$ and $\mathbf{x}$. In the factor market, the price and quantity variables are w (wage) and c (consumption level).[1] Consumption is $\mathbf{y} = \mathbf{f}c$, where $\mathbf{f}$ represents the proportions of consumption. The factor suppliers (households) have the following pair of complementarities. Excess factor supply N means a zero price w: $0 \leq w \perp N - \mathbf{lx} \geq 0$, where $\mathbf{l}$ is the row vector of factor input coefficients, so that factor costs are $\mathbf{v} = w\mathbf{l}$. And a negative budget means zero consumption: $0 \leq c \perp \mathbf{pf}c - wN \geq 0$.

The variables are quantities and prices $\mathbf{z} = (\mathbf{x}, c, w, \mathbf{p})$ and they are complementary to losses in production and consumption and to excess supplies of factor inputs and outputs $\mathbf{\Phi}(\mathbf{z}) = (w\mathbf{l}-\mathbf{p}(\mathbf{I}-\mathbf{A}), \mathbf{pf}c - wN, N - \mathbf{lx}, (\mathbf{I}-\mathbf{A})\mathbf{x}-\mathbf{f}c)$: $\mathbf{0} \leq \mathbf{z} \perp \mathbf{\Phi}(\mathbf{z}) \geq \mathbf{0}$. By nonnegativity of the components of $\mathbf{z}$ and $\mathbf{\Phi}(\mathbf{z})$, this economy-wide complemen-

[1] More factors will be entered below. The implicit assumption of the use of consumption level to measure utility is that the latter is a Leontief, fixed-proportions function.

tarity is equivalent to component-by-component complementarity. The parameters which are no variables define the *structure* of the economy, that is, $(\mathbf{l}, \mathbf{A}, \mathbf{f}, N)$. These are the input coefficients of production and consumption and the endowment of the economy. The generalized loss function $\boldsymbol{\Phi}$ is parametrized by the structure of the economy. An equilibrium is a solution to the complementarity problem $\mathbf{0} \leq \mathbf{z} \perp \boldsymbol{\Phi}(\mathbf{z}) \geq \mathbf{0}$ (Ferris and Pang, 1997). As different economies have different equilibria, an equilibrium is a function of the loss function: $\mathbf{z}^* = \mathbf{Z}(\Phi)$. If the economy is parametrized by its input–output structure, we may write $\mathbf{z}^* = \mathbf{Z}(\mathbf{l}, \mathbf{A}, f, N)$.

3. STOCHASTIC ANALYSIS

Stochastic analysis studies the transmission of uncertainty in the structure of the economy, $\boldsymbol{\Phi}$, possibly parametrized by input–output structure $(\mathbf{l}, \mathbf{A}, \mathbf{f}, N)$, to the equilibrium. The uncertainty is given by a distribution of the structure, $\boldsymbol{\Phi} \sim F$. Then $\mathbf{z}^* \sim G = F(\mathbf{Z}^{-1})$. There is no need to invert reduced form $\mathbf{Z}$ of the solution, it suffices to take Monte Carlo drawings from F, to determine the respective equilibrium values, and to build their distribution, G. Although each Monte Carlo iteration involves solving a big model, this methodology is feasible because the nonlinear complementarity problem routine of GAMS takes extremely little time computing the equilibria of the draws.

If the structure of the economy is parametrized, distribution F has components, $F_{\mathbf{l}}, F_{\mathbf{A}}, F_{\mathbf{f}}, F_N$. Ideally, these components are derived from micro data (Mattey and ten Raa, 1997; ten Raa and Rueda-Cantuche, 2007), but the uncertainty may be a subjective summary of the uncertainty that surrounds a policy, as we illustrate now by extension to environmental economics.

Analogous to the factor input coefficients $\mathbf{l}$ and the endowment N, include emission coefficients $\mathbf{m}$ and a pollution cap M in the structure of the economy. Then, the variables and the model become $\mathbf{0} \leq \mathbf{z} = (\mathbf{x}, c, w, t, \mathbf{p}) \perp \boldsymbol{\Phi}(\mathbf{z}) = (w\mathbf{l} + t\mathbf{m} - \mathbf{p}(\mathbf{I} - \mathbf{A}),\ \mathbf{pf}c - wN - t\mathbf{M}, N - \mathbf{lx}, \mathbf{M} - \mathbf{mx}, (\mathbf{I} - \mathbf{A})\mathbf{x} - \mathbf{f}c) \geq \mathbf{0}$. Similar inclusion of capital coefficients and endowment, aggregation of all but the last components of $\mathbf{z}$ and $\boldsymbol{\Phi}(\mathbf{z})$ by national economy i (= Spain, Denmark or Belgium), replacement of the last component of $\boldsymbol{\Phi}(\mathbf{z})$ by the same expression but summed over national economies, and a minor modification to accommodate trade imbalances yields the model of ten Raa and Shestalova (2014). The pollution caps M^i for each national economy were taken from the Kyoto agreement (reductions of base-year emissions).

While in principle the allowed environmental damage levels are exogenously determined by international treaties, problems of implementation or uncertainty about the propagation of the environmental changes may lead us to treat them as random parameters. The environmental economics literature stresses the presence of uncertainty both about the measurement of emission as about their effects. For example, with respect to greenhouse gases, uncertainty is large for non-CO_2 greenhouse emission trends and abatement options (e.g. Gielen and Kram, 1998). In addition, there is also uncertainty regarding the effects of emissions on environment (Manne et al., 2004). Therefore, the illustrative example provided below focuses on the environmental uncertainty, ignoring the uncertainty that surrounds the input and output data. We assume the distributions of the emission caps (one for each country) to be one-sided (implementation problems may yield underachievements) and uniform (there are few observations hence the least informative distribution is the natural prior): $F^i_M = (M - M^i)/(M^i\varepsilon^i)$ on $M^i \le M \le M^i(1+\varepsilon^i)$. We assume $\varepsilon^i = 0.5$ (50%), and compute statistics based on 100 runs of the model.

The most interesting components of solution **z** are the expansion factors, $c^i \sim G_{c^i}$. They are dependent, but it is straightforward to compute their marginal distributions and, in particular, their means and 95% confidence intervals.

4. ILLUSTRATION

To demonstrate the applicability of the approach, we extend the illustrative example from our earlier paper (ten Raa and Shestalova, 2014) to the case of uncertainty about environmental effects. The example features three developed economies, namely Belgium, Denmark and Spain, as their data sets were completed and made available to us. For the sake of simplicity, we restrict attention to the main greenhouse gas, CO_2, which accounts for about 80% of greenhouse pollution of the EU-15 (Gielen and Kram, 1998). However, the extension to an interregional model of all EU member states and more pollutants is straightforward once the respective data become available.

We first describe the data and then compare the model outcomes in the deterministic case to those in the stochastic case. The data description section and the deterministic case are drawn from ten Raa and Shestalova (2014).

4.1. Data

The data, provided to us by the Institute for Prospective Technological Studies (IPTS), are based on the national and environmental accounts of each country, published by Eurostat for the year 2000. The use and supply tables, **U** and **V**, incorporate 59 products and industries. The tables are expressed in producer prices. The supply tables of the Eurostat are published in producer prices, while the use tables were estimated by the IPTS. For Denmark, the data have been converted into euros using the average exchange rate of 2000.[2] We treat the commodities with negligible values of the observed net export as non-tradable.[3] The trade with the rest of the world has been held fixed at the observed level.

Labour data (number of employed persons) originate from the EU KLEMS data (O'Mahony and Timmer, 2009). Labour unemployment rates in 2000 were as follows: Belgium 6.9%; Denmark 4.3%; Spain 11.1% (Eurostat, 2010). These percentages represent "unemployed persons as a percentage of the labor force, where the labor force is the total number of people employed and unemployed". The total private capital stock in each country comes from the Kiel Institute of the World Economy's Database on Capital Stocks in OECD countries (Kamps, 2006). As usual, capital employment data are a bottleneck. Since no detailed capital employment data were available by industry (in

[2] Since we consider the aggregated production technology for each industry, the analysis does not account for within industry reallocations and environmental improvements.

[3] In this application, non-tradable commodities are as follows: secondary metals, water, sewage, transportation, real estate, education, health care, public services, services of membership organization and private households, and other services.

TABLE 1. Effect of environment policy targets on expansion factors under international trade in CO_2-emission permits and the effects of uncertainty.

Consumption levels	Spain	Denmark	Belgium
Emission limits	0.85	0.79	0.86
Unconstrained emissions	1.049	1.133	1.115
With caps and permits trade	0.903	1.015	1.094
One-sided uncertainty	1.031	1.134	1.105
95% confidence interval	(1.025, 1.038)	(1.127, 1.141)	(1.098, 1.112)

our industry classification), capital employment has been allocated across industries proportionally to the industrial gross operating surpluses, assuming capacity utilization rates around 0.82, in accordance with the EU average utilization rates for that period (World Bank, 2010).

The environmental impact data come from the database of the IPTS, that builds upon the environmental accounts published by the Eurostat. We focus on CO_2-emissions and assume the policy parameters, emission limits μ^i, to reflect the respective Kyoto targets on the reduction of greenhouse gas emissions.[4] The original Kyoto targets (-7.5% for Belgium, and -21% for Denmark, and $+15\%$ for Spain) specify the percentage of the emission reduction compared with the level of 1990, while our data correspond to 2000. Therefore, we have transformed them by the following formula: $\mu_{2000} = \mu_{1990}/(1+g)$, in which the subscripts refer to the two base years, and g denotes the change in the CO_2 emission level between these years. The data on g are derived from EEA (2002): 7.7% for Belgium, -0.4% for Denmark, and 34.9% for Spain. Plugging these values into the 1990–2000 transformation formula above yields the values of the Kyoto reduction factors: emission limits μ^i.

4.2. Results

In Table 1, the first line shows emission limits μ^i, the second line the unconstrained equilibrium consumption values – without the emission caps – and the third line the equilibrium consumption values with caps and international trade in CO_2-emission permits in the deterministic case. These lines are taken from ten Raa and Shestalova (2014). The fourth line shows the consumption levels under one-sided uncertainty, and the fifth line the 95% confidence interval.

As can be seen from Table 1, a 50%-uncertainty about the realization of μ's results in a relatively small variance of the resulting expansion factors.

[4] The Kyoto targets cover emissions of the six main greenhouse gases, namely carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons (HFCs), perfluorocarbons (PFCs), and sulphur hexafluoride (SF_6). The total value of greenhouse gas emissions is constructed as a weighted average of each gas emissions with global warming potentials as weights. The targets specify the percentage of reductions of the total emission level to be achieved in the period 2008–2012 compared with the level of 1990. In addition to these national caps, there is the EU trading scheme currently covering only CO_2 emissions of major installations. Not all industries are covered by the EU emission trading scheme.

5. CONCLUSION

The complementarity between the quantity and value systems of input–output analysis is shown to be the basis of the complementarity problem approach to computable general equilibrium. Environmental regulatory constraints are incorporated in the same vein as primary input constraints. The numerical illustration demonstrates the applicability of the model to stochastic analysis of input–output scenarios under uncertainty concerning the measurement of environmental changes and their effects.

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