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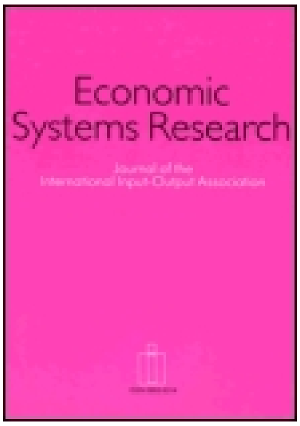

Economic Systems Research
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/cesr20

The Construction of Input-Output Coefficients Matrices in an Axiomatic Context: Some Further Considerations

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Available online: 22 Sep 2010

To cite this article: Thijs ten Raa & José Manuel Rueda-Cantuche (2003): The Construction of Input-Output Coefficients Matrices in an Axiomatic Context: Some Further Considerations, Economic Systems Research, 15:4, 439-455

To link to this article: http://dx.doi.org/10.1080/0953531032000152317

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Economic Systems Research, Vol. 15, No. 4, December 2003

Routledge
Taylor & Francis Group

The Construction of Input–Output Coefficients Matrices in an Axiomatic Context: Some Further Considerations

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(*Received August 2002; revised March 2003*)

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ABSTRACT *Kop Jansen & ten Raa (1990) established a purely theoretical solution to the problem of selecting a model for the construction of coefficients on the basis of make and use tables. In an axiomatic context, they singled out the so-called commodity technology model as the best one according to some desirable properties. The aim of this paper is to delineate the restrictions on the relevant data sets that ensure fulfilment of the desirable properties by other models used by statistical offices.*

KEYWORDS: *Make and use tables; technical coefficients*

1. Introduction

An input–output matrix of technical coefficients, $\mathbf{A} = (a_{ij})$ with $i, j = 1, \ldots, n$ (where n is the number of commodities), represents the direct requirements of commodity i needed to produce a physical unit of commodity j. For instance, if industry 1 corresponds to agriculture and industry 2 corresponds to chemicals, then a_{21} will be the amount of chemical products consumed by agriculture per physical unit of peach, apple and so on. In more general terms, the standard reference is Leontief (1986).

The matrix of technical coefficients $\mathbf{A}$ has been used for economic analysis by means of the so-called quantity equation or material balance (supply and demand) and the value equation or financial balance (costs and revenues),

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}$$

$$\mathbf{p} = \mathbf{p}\mathbf{A} + \mathbf{v}$$

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ISSN 0953-5314 print; 1469-5758 online/03/040439-17 DOI: 10.1080/0953531032000152317
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Here, $\mathbf{x}$ is a column vector of gross outputs, $\mathbf{y}$ is another column vector of final demand, $\mathbf{p}$ is a row vector of prices, and lastly, $\mathbf{v}$ is a row vector of value-added coefficients.

The quantity equation is used for national or regional economic planning; for instance, the output requirements to satisfy a certain final demand level could be analysed. Final demand could be influenced by an exports or investments policy. Thus, there will be a direct effect over the output levels, which will depend on the final demand variations ($\Delta\mathbf{y}$) and additional indirect effects that will be determined by the $\mathbf{A}$-matrix, in accordance with the material balance equation. The value equation can be used to assess the price effects resulting from an energy shock, which surely will bring about variations in the value-added shares of a product, to mention an example.

National and Regional Statistical Offices have concentrated almost exclusively on industry input–output tables instead of commodity tables and set up so-called transactions tables $\mathbf{T} = (t_{ij})$, with $i,j = 1, \ldots, n+1$, where n is the number of sectors or industries (ten Raa, 1994). In such a table, each element displays the input requirements of sector i per unit of sector j's production, as well as the final demand compartments (household and government consumption, investment and net exports).

Ten Raa (1994) noted that an input–output transactions table $\mathbf{T}$ reduces the construction of a matrix of technical coefficients $\mathbf{A}$ to a matter of divisions:

$$a_{ij} = \frac{t_{ij}}{\sum_{s=1}^{n+1} t_{js}}$$

However, there are three problems. First, commodities and sectors cannot always be classified in the same way. Second, in addition to a multitude of inputs, sectors may also have a multitude of outputs. In other words, secondary products must be accommodated. Third, commodities contained in each row and column of an industry-by-industry table are not homogeneous in terms of production (see Rainer, 1989; Braibant, 2002).

To address these complications, the System of National Accounts proposed by the United Nations (1968, 1993), first established the concepts of use and make matrices within an input–output framework.[1] Demand and supply of commodities are described by industries. Thus, let us define a use table, $\mathbf{U} = (u_{ij})$ of commodities i consumed by sector j (with $i,j = 1, \ldots, n$), and a make table $\mathbf{V} = (v_{ij})$ where sector i will produce commodity j (United Nations, 1968; ten Raa *et al.*, 1984; Kop Jansen & ten Raa, 1990). Notice that, although several attempts have been made to deal with rectangular use and make matrices (see Konijn, 1994), the numbers of commodities and of industries are presumed equal. Following Kop Jansen & ten Raa (1990) industry tables and mixed tables are not considered either.

This new framework provided a more accurate description of commodity flows and, at the same time, made economists face the new problem of constructing technical coefficients matrices, according to some mathematical method based on use and make matrices, and which did not always make sense economically (Viet, 1994).

Basically, the construction of a technical coefficients matrix $\mathbf{A}$ is a matter of treatment of secondary products. Many establishments produce only one group of commodities, which are the primary products of the industry to which they are

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Table 1. General input–output accounting framework

	Commodities	Industries	Final demand	Total
Commodities		$\mathbf{U}$	$\mathbf{Y}$	$\mathbf{q}=\mathbf{e}^{T}\mathbf{V}$
Industries	$\mathbf{V}$			$\mathbf{g}=\mathbf{Ve}$
Primary inputs		$\mathbf{W}^{T}$		
Total	$\mathbf{V}^{T}\mathbf{e}$	$\mathbf{e}^{T}\mathbf{V}^{T}$		

classified. However, some establishments produce commodities that are not among the primary products of the industry to which they belong. As a result, non-zero off-diagonal elements appear in the make matrix. Alternative treatments of secondary products rest upon the separation of outputs and inputs associated with secondary products so that they can be added to the outputs and inputs of the industry in which the secondary product is a characteristic output. Assumptions on these input structures imply an **A**-matrix of technical coefficients as a function of the use and make matrices.

In what follows, **e** will denote a column vector with all entries equal to one, ^{T} will denote transposition and $^{-1}$ an inversion of a matrix. Since the latter two operations commute, their composition may be denoted $^{-T}$. In addition, $\hat{}$ will denote diagonalization, whether by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector. $\tilde{}$ will denote a matrix with all the diagonal elements set to null. Table 1 shows the general input–output accounting framework from the SNA (see United Nations, 1968, 1973; Gigantes, 1970; Armstrong, 1975).

Section 2 reviews the literature on the different methods for the treatment of secondary products. In Section 3, we show the theoretical solution given by Kop Jansen & ten Raa (1990) in order to select the best method for constructing a technical coefficients matrix $\mathbf{A} = (a_{ij})$ (of commodities i needed for the production of one physical unit of commodity j). Further considerations will be taken into account with respect to hybrid models. The aim of this paper is to analyse how bad or good are alternative methods of constructing technical coefficients' **A**-matrices in the presence of data restrictions; this is done in Section 4. Lastly, Section 5 draws some conclusions.

2. Description of the Models

Table 2 describes the literature on the treatment of secondary products. The methods can be divided into two groups: those which transfer outputs only and those which transfer both inputs and outputs.

2.1. Methods based on the Transfer of Outputs Only

Methods based on the transfer of outputs only are not based on economic assumptions, but are mainly statistical devices to remove secondary products from the make table. Viet (1994) suggests that the Stone and ESA methods should be used only for by-products.[2]

2.1.1. Transfer Method The transfer method treats a secondary product as if it is sold by the industry to which it is a primary product to the industry that actually

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Table 2. Treatment of secondary products

Review of approaches to the treatment of secondary products
1. Transfer of outputs alone:
(a) *Transfer method* (Stone, 1961, pp. 39–41; United Nations, 1973, p. 25; Fukui & Seneta, 1985, p. 178; Viet, 1986, pp. 16–18; Kop Jansen & ten Raa, 1990, p. 215; or Viet, 1994, pp. 36–38).
(b) *Stone method or by-product technology model* (Stone, 1961, pp. 39–41; United Nations, 1973, p. 26; ten Raa *et al.*, 1984, p. 88; Fukui & Seneta, 1985, p. 178; Viet, 1986, pp. 15–16; Kop Jansen & ten Raa, 1990, p. 215; or Viet, 1994, p. 38).
(c) *European System of Integrated Economic Accounts* (*ESA*) *method* (EUROSTAT, 1979; Viet, 1986, pp. 18–19; Kop Jansen & ten Raa, 1990, p. 214; or Viet, 1994, pp. 38–40).
2. Transfer of inputs and outputs:
2.1 *Lump-sum or aggregation method* (Office of Statistical Standards, 1974, p. 116; Fukui & Seneta, 1985, p. 177; Kop Jansen & ten Raa, 1990, p. 214; or Viet, 1994, pp. 42–43).
2.2 *One technology assumption methods*
(a) *Commodity technology model* (United Nations, 1968, 1973 pp. 26–32; van Rijckeghem, 1967; Gigantes, 1970, pp. 280–284; Armstrong, 1975, pp. 71–72; ten Raa *et al.*, 1984, p. 88; Viet, 1986, p. 20; Kop Jansen & ten Raa, 1990, p. 215; or Viet, 1994, p. 41).
(b) *Industry technology model* (United Nations, 1968, 1973 pp. 26–32; Gigantes, 1970, pp. 272–280; Armstrong, 1975, pp. 71–72; ten Raa *et al.*, 1984, pp. 88–89; Fukui & Seneta, 1985, p. 178; Viet, 1986, p. 21; Kop Jansen & ten Raa, 1990, p. 215; or Viet, 1994, pp. 40–41).
(c) *Activity technology model* (Konijn, 1994; Konijn & Steenge, 1995).
2.3 *Hybrid technology assumptions methods*
(a) *Mixed commodity and industry technology assumptions* (United Nations, 1968, 1973, pp. 33–34; Gigantes, 1970, pp. 284–290; Armstrong, 1975, pp. 72–76).
(b) *Ten Raa et al.*, 1984 (Commodity technology assumption and by-product technology method).

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produces it. Mathematically, the technical coefficients matrix $\mathbf{A}$ is derived by the following formula, where $\mathbf{g} = \mathbf{Ve}$ and $\mathbf{q} = \mathbf{e}^{\mathrm{T}}\mathbf{V}$ (see Table 1):

$$\mathbf{A}_{\mathrm{T}}(\mathbf{U}, \mathbf{V}) = (\mathbf{U} + \tilde{\mathbf{V}})(\hat{\mathbf{g}} + \hat{\mathbf{q}} - \hat{\mathbf{V}})^{-1}$$

The input structure of the industry to which the secondary products are primary outputs, is distorted by the inclusion of the transfer. As a result, an increase in the final demand of those secondary products would lead to an increase in the demand for the primary outputs of the industry that actually produces them, which need not be true. In addition, the input structures of industries that produce secondary products can be altered if the proportion in which they are produced changes. Lastly, sector outputs can either be industry or commodity outputs.

2.1.2. Stone Method (or By-product Method) By the Stone method, all secondary products are considered by-products. Therefore, they can be treated as a negative input in the industry where it is actually produced. Mathematically, we can obtain the technical coefficients matrix $\mathbf{A}$ by the following formula:

$$a_{ij}^{B}(\mathbf{U}, \mathbf{V}) = \begin{cases} \dfrac{u_{jj}}{v_{jj}} & \text{if } i = j \\[2ex] \dfrac{u_{ij} - v_{ji}}{v_{jj}} & \text{if } i \neq j \end{cases}$$

or, in matrix notation,

$$\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V})=(\mathbf{U}-\hat{\mathbf{V}}^{\mathrm{T}})\hat{\mathbf{V}}^{-1}$$

Negative values of technical coefficients are obtained as a result of applying the Stone method. They appear when the actual use of a commodity i by an industry j is smaller than its secondary output of commodity i. Industry j would need a net amount $u_{ij}-v_{ji}$ of commodity i, which is actually a secondary product of industry j, for the production of v_{jj} units of its primary output. Note that, necessarily, $i\neq j$. Besides, the input structure of the industry that produces the secondary product is distorted according to this method for the treatment of secondary outputs.

2.1.3. ESA Method The European System of Integrated Economic Accounts (ESA) published in 1979 recommends that secondary products should be treated as if they were produced by the industry for which these secondary outputs are primary products. Mathematically, the technical coefficients matrix **A** is calculated as follows:

$$a_{ij}^{E}(\mathbf{U},\mathbf{V})=\frac{u_{ij}}{\sum_{j=1}^{n}v_{ji}}\qquad i,j=1,2,\ldots,n$$

or, in matrix notation,

$$\mathbf{A}_{\mathrm{E}}(\mathbf{U},\mathbf{V})=\mathbf{U}\hat{\mathbf{q}}^{-1}$$

The technical coefficients are constructed by division of all the entries of the use table by the total output of the commodity corresponding to the column in the make table. This total output is not necessarily produced by a single sector.

The shortcoming of this treatment is the distortion of input structures of industries with no secondary products but with primary products, which are also produced by other industries. Input structures of industries with secondary production are distorted similarly. Moreover, the sum of intermediate uses of an industry j can be larger than the total output of commodity j due to the subtraction of their secondary outputs. This will lead to sums of input coefficients greater than one and, therefore, to the non-existence of the Leontief inverse or the negativity of some of its cells.

2.2. Methods based on the Transfer of Inputs and Outputs

Other methods transfer secondary products and their inputs to the outputs and inputs of the industries where the secondary products constitute a primary output. Since data on inputs associated with secondary products are rarely available, assumptions are made, such as the commodity technology hypothesis, by which the input structure of a secondary commodity is independent of the sector of production, or the industry technology hypothesis, by which the input structure of a secondary output is the same as that of the industry to which it is a primary output. In other terms, production processes substitute commodities when the activity technology model is applied (Konijn, 1994). In fact, this model borrows the mathematical structure of the commodity technology model. In addition, we will present hybrid methods based on mixed technology assumptions.

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2.2.1. Lump-sum or Aggregation Method This method treats a secondary product as if it were actually produced as a primary output. Mathematically, the matrix of technical coefficients **A** is given by:

$$a_{ij}^{L}(\mathbf{U},\mathbf{V}) = \frac{u_{ij}}{\sum_{i=1}^{n} v_{ji}} \qquad i,j = 1,2,\ldots,n$$

or, in matrix notation,

$$\mathbf{A}_{\mathrm{L}}(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}$$

Technical coefficients are obtained by dividing all the entries of each of the columns from the use table by the total output of industry *j*, specified in row *j* of the make table. This total output includes not only primary products, but also secondary products and by-products.

2.2.2. Methods with a Single Technology Assumption Three methods rely on a single technology assumption, namely the commodity, industry and activity technology models.

The commodity technology model assumes that each commodity has its own input structure, irrespective the industry of production.[3] Hence, if a_{ik} represents the direct requirements of commodity *i* needed by industry *j* for the production of one physical unit of commodity *k* and also v_{jk} stands for the total secondary output of commodity *k* produced by industry *j*, it can be derived that the amount $a_{ik}v_{jk}$ is the total inputs requirements of commodity *i* needed for the production of v_{jk} units of output *k*. Then, if we also assume that industry *j* has multiple outputs and they are all different from commodity *k*, we could finally sum over outputs to obtain industry *j*'s total demand for input *i*. Thus, u_{ij} can be written as:

$$u_{ij} = \sum_{k=1}^{n} a_{ik} v_{jk} \qquad i,j = 1,\ldots,n$$

If industry *j* produces v_{jk} outputs of commodity *k*, a_{ik} inputs of commodity *i* per physical unit of output *k* will be required. Furthermore, in case another industry *t* produces v_{tk} outputs of commodity *k*, the direct requirements of input *i* per physical unit of output *k* result again a_{ik}. In matrix terms, the commodity technology assumption is given by:

$$\mathbf{U} = \mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}$$

and therefore,

$$\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{V}^{-\mathrm{T}}$$

This method requires the same number of commodities as of industries due to the inverse of the make table. However, its main shortcoming is the negativity of some technical coefficients. According to Viet (1994), negative elements arise when the input structure of the secondary output is not the same as that of the primary product produced elsewhere, and the input, which is transferred out, is greater than the input that is actually consumed. Negative values in the **A**-matrix have prompted a huge literature on the possible solutions to overcome this shortcoming (Almon, 1970; Armstrong, 1975; ten Raa *et al.*, 1984; ten Raa, 1988; ten Raa & van der Ploeg, 1989; Rainer, 1989; Steenge, 1990; Rainer & Richter, 1992; Mattey,

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1993; Konijn, 1994; Konijn & Steenge, 1995; Mattey & ten Raa, 1997; Avonds & Gilot, 2002).

By the industry technology assumption, each industry has the same inputs requirements for any unit of output (this time, measured in values). This implies that every commodity has different technologies depending on what industry produces it. Actually, the industry technology model assumes that:

(1) Input structures of industries are proportional to their outputs (as the commodity technology model assumes).
(2) Market shares of industries are fixed and independent of the level of commodity or industry outputs.

Mathematically, the **A**-matrix of technical coefficients is given by:

$$a^I_{ij}(\mathbf{U},\mathbf{V}) = \sum_{k=1}^{n}\left(\frac{u_{ik}}{v_k}\right)\left(\frac{v_{kj}}{v'_j}\right)$$

where v_k is the total output of industry k and v'_j is the total output of commodity j. In matrix notation,

$$\mathbf{A}_{\mathrm{I}}(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}$$

Let us examine in more detail the above expression in order to cast light on the economic foundations of the industry technology model. u_{ik}/v_k represents the direct requirements of commodity i needed for the production of one physical unit of commodity k. On the contrary, v_{kj}/v'_j denotes the proportion of the commodity j output produced by industry k to the total output of such a commodity. In short, the result is called market share. Hence, according to this model, technical coefficients result from a (market share) weighted average over industries k.

Although this assumption has been widely used in many countries, its popularity stemming from the non-negativity of the resulting technical coefficients matrix as well as the fact that the number of commodities need not be equal to the number of industries, is economically unacceptable. As Viet (1994) pointed out, the resulting **A**-matrix is obtained assuming that costs associated with either primary or secondary products are the same, while prices of these products are obviously different. Since the financial balance in input–output analysis states that for each commodity unit, revenue equals material cost plus value added, applying the industry technology model implies that this meaningful input–output economics assumption does not hold. Moreover, a change in the base year prices and also proportional variations in input requirements and outputs alone will affect the internal structure of technical coefficients.

With respect to the activity technology model, Konijn (1994) redefines the use and make matrices in such a way that application of the commodity technology formula prompts no negatives. Instead of the commodity unit, Konijn assumes that industries can produce commodities according to several production processes and that the same production process can be used by other industries. Moreover, one commodity can be produced by several production processes, while one production process can generate different goods. Unfortunately, the resulting activity-by-activity input–output table does not remove the negatives. Konijn (1994) and Konijn & Steenge (1995) argue that the remaining negatives indicate that some classification adjustments must be made or some further research on error data must be developed. Although the need for further information on the use

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and make system is required to apply this activity technology model, Statistics Netherlands adopts this way of removing negatives. In conclusion, Konijn (1994) proposes that we explicitly look at production processes instead of commodities and that we consider the commodity classification of use and make matrices an instrument instead of an exogenous scheme.

2.2.3. Mixed Technology Assumptions Methods Following Armstrong (1975), hybrid technology methods assume that subsidiary production fits either the commodity technology or the industry technology assumptions. That is, one would expect that most commodities have the same input structure wherever they are produced, but when secondary products are obtained as a result of industrial processes (i.e. by-products), the assumption of an industry technology assumption may be more appropriate. Hybrid methods require that the make matrix is split into two matrices, $\mathbf{V}_1$ and $\mathbf{V}_2$, where the first one includes outputs for which the commodity technology assumption is made and the second includes those which are to be treated on an industry technology assumption.

The hybrid technology methods suggested by Gigantes (1970), and incorporated in the United Nations System of National Accounts (United Nations, 1968), are based on the following assumptions over the use and make matrices.

Firstly, industries' outputs of commodities for which a commodity technology assumption is made, are proportional to the output of each industry. This is implicitly assumed when the commodity technology model is applied. It is denoted as:

$$\mathbf{V}_1^{\mathrm{T}} = \mathbf{C}_1\hat{\mathbf{g}}_1 \tag{1}$$

where $\mathbf{g}_1 = \mathbf{V}_1\mathbf{e}$.

Secondly, industries' outputs of commodities for which an industry technology assumption is made, are proportional to these commodity outputs. The proportions are the market shares of each industry's products in the total output of each one of these kinds of commodities. This is implicitly assumed when the industry technology model is applied:

$$\mathbf{V}_2 = \mathbf{D}_2^{\star}\hat{\mathbf{q}}_2 \tag{2}$$

where $\mathbf{q}_2 = \mathbf{V}_2^{\mathrm{T}}\mathbf{e}$.

Thirdly, the production of commodities for which an industry technology assumption is made follows fixed market shares. That is, industries' commodity outputs for which an industry technology assumption is made are proportional to the total commodity outputs produced in the economy. It is denoted as:

$$\mathbf{g}_2 = \mathbf{D}_2\mathbf{q} \tag{3}$$

where $\mathbf{g}_2 = \mathbf{V}_2\mathbf{e}$.

Following Armstrong (1975), after some transformations the resulting matrix of technical coefficients is given by:

$$\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{I}-\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2)+\mathbf{V}_2\hat{\mathbf{q}}^{-1})$$

It can be seen that if $\mathbf{V}_2 = 0$ then $\mathbf{V} = \mathbf{V}_1$, $\mathbf{g}_1 = \mathbf{g}$ and $\mathbf{q}_2 = 0$; hence, the solution is given by:

$$\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V}) = \mathbf{U}\mathbf{V}^{-\mathrm{T}} = \mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})$$

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which is the commodity technology model. Analogously, if $\mathbf{V}_1 = 0$ then $\mathbf{V} = \mathbf{V}_2$, $\mathbf{q}_2 = \mathbf{q}$ and $\mathbf{q}_1 = 0$; hence the solution is given by:

$$\mathbf{A}_{\mathrm{H}}(\mathbf{U}, \mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1} = \mathbf{A}_{\mathrm{I}}(\mathbf{U}, \mathbf{V})$$

which is the industry technology model.

The hybrid technology model generates negative values of technical coefficients, just like the commodity technology model. A different solution can be obtained if we assume that the outputs of the products for which an industry technology assumption is made are proportional to the outputs of the producing industries instead of to the outputs of each commodity, whatever industry produced them. A more detailed explanation is shown in Armstrong (1975, pp. 74–76). The resulting **A**-matrix is given by:

$$\mathbf{A}_{\mathrm{Y}}(\mathbf{U}, \mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{I} - \mathbf{V}_2^{\mathrm{T}}\hat{\mathbf{g}}^{-1}\mathbf{H}) + \mathbf{V}_2\hat{\mathbf{q}}^{-1})$$

with **H** such that $\mathbf{g} = \mathbf{H}\mathbf{q}$.

Ten Raa *et al.* (1984) elaborated a new hybrid technology model where the industry technology assumption was replaced by the Stone (or by-product) method. In this case, the make table is split into two matrices, $\mathbf{V}_1$ and $\mathbf{V}_2$. $\mathbf{V}_1$ includes those outputs for which a commodity technology assumption is made (primary and ordinary secondary outputs) and $\mathbf{V}_2$ includes those that are to be treated on by-product technology assumption (by-products). Since by-products are treated as negative inputs, the total requirements of commodity i by industry j for the production of all primary and ordinary secondary products of industry j are given by a net amount of requirements. Mathematically,

$$\mathbf{A}_{\mathrm{CB}}(\mathbf{U}, \mathbf{V}) = (\mathbf{U} - \mathbf{V}_2^{\mathrm{T}})\mathbf{V}_1^{-\mathrm{T}}$$

Notice that if all secondary products are ordinary ($\mathbf{V}_1 = \mathbf{V}$), we will have the commodity technology model and that if all secondary products are by-products ($\mathbf{V}_1 = \hat{\mathbf{V}}, \mathbf{V}_2 = \tilde{\mathbf{V}}$), we will have the Stone method (or by-product technology model). As discussed in ten Raa *et al.* (1984), negative elements in the technical coefficients matrix also arise when this hybrid model is applied.

3. The Choice of Model

Kop Jansen & ten Raa (1990) developed and examined axiomatically how well various methods for treatment of secondary products fulfil four desirable properties of input–output coefficients $\mathbf{A}(\mathbf{U}, \mathbf{V})$, namely:

(1) Axiom M refers to the material balance or the quantity equation and is denoted by

$$\mathbf{A}(\mathbf{U}, \mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} = \mathbf{U}\mathbf{e}$$

Economically, this axiom implies that total supply must meet total demand (intermediate consumption plus final demand compartments). In other words, the total input requirements must be equal to the observed total input.

(2) Axiom F refers to the financial balance or the value equation and is denoted by

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U}, \mathbf{V})\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

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That is to say, for each commodity unit, revenue equals material cost plus value added. In words, the input cost of output must match the observed value of input.

(3) Axiom P refers to the invariance of the resulting **A**-matrix with respect to units of measurement or, in other terms, to prices. It is called the price invariance axiom and is denoted by

$$\mathbf{A}(\hat{\mathbf{p}}\mathbf{U}, \mathbf{V}\hat{\mathbf{p}}) = \hat{\mathbf{p}}\mathbf{A}(\mathbf{U}, \mathbf{V})\hat{\mathbf{p}}^{-1}\ \forall \mathbf{p} > 0$$

Evidently, this property tries to avoid that a change in the base year prices could affect technical coefficients. Variations in the internal structure of $\mathbf{A}(\mathbf{U}, \mathbf{V})$ should be caused by real economic phenomena.

(4) Axiom S is the so-called scale invariance axiom:

$$\mathbf{A}(\mathbf{U}\hat{\mathbf{s}}, \hat{\mathbf{s}}\mathbf{V}) = \mathbf{A}(\mathbf{U}, \mathbf{V})\ \forall \mathbf{s} > 0$$

It stipulates that technical coefficients do not change when input requirements and outputs vary proportionally.

Kop Jansen & ten Raa (1990) proved that the just described structure of input–output analysis, involving the four axioms, not only imposes restrictions on the choice of model of construction, but determines it uniquely, namely the commodity technology model. Their theorem in the real sphere states that the combination of axioms M and S implies that the commodity technology model specifies the **A**-matrix, and their theorem in the nominal sphere that the combination of axioms F and P has the same implication.

Table 3 illustrates how the other methods fare in the light of the axioms. In addition to the results obtained by Kop Jansen & ten Raa (1990), Table 3 includes the performance of the mixed commodity and industry technology models (United Nations, 1968; Gigantes, 1970); the proofs are in the Appendix. Lastly, since the activity technology model (Konijn, 1994) borrows the mathematical structure from the commodity technology model, it requires no separate performance report.

Our main result is a closer examination of Table 3, answering the question of what restrictions on the data restore the desirable properties for the models. In other words, for each input–output construct we delineate regions in data space where axioms are fulfilled.

4. Equivalent Conditions for Axioms M and F

In this section, we will prove that under certain data limitations, some methods for the treatment of secondary products other than the commodity technology model

Table 3. Axioms fulfilment of input–output coefficients constructs

Model	Axiom M	Axiom F	Axiom P	Axiom S
Transfer				
Stone method			✓	✓
ESA method	✓		✓	
Lump-sum				✓
Commodity technology	✓	✓	✓	✓
Industry technology	✓			
ten Raa *et al.* Method			✓	✓
United Nations hybrid model	✓			

Source: Kop Jansen & ten Raa (1990) and own elaboration.

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fulfil the material and financial balance. Unfortunately, the price and scale invariance axioms admit no such results.

Two theorems express the material and financial balance axioms in terms of the commodity technology model.

Theorem 1

A technical coefficients matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$ fulfils axiom M for all $\mathbf{U}$ and non-singular $\mathbf{V}$ if and only if,

$$\sum_{j=1}^{n} a_{ij}q_j = \sum_{j=1}^{n} a_{ij}^{C}q_j \qquad \forall i = 1, \ldots, n$$

or, in matrix terms,

$$\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} = \mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}$$

Proof

By definition of $\mathbf{A}_{\mathrm{C}}$, the right-hand side of axiom M reads

$$\mathbf{U}\mathbf{e} = \mathbf{U}\mathbf{V}^{-\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{e} = \mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}$$

This completes the proof.

Theorem 2

A technical coefficients matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$ fulfils axiom F if and only if the sum of each column of $\mathbf{A}(\mathbf{U},\mathbf{V})$ matches the sum of the respective column of $\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})$ for all $\mathbf{U}$ and non-singular $\mathbf{V}$.

$$\sum_{i=1}^{n} a_{ij} = \sum_{i=1}^{n} a_{ij}^{C} \qquad \forall j = 1, \ldots, n$$

or, in matrix terms,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})$$

Proof

Sufficiency is proved as follows. Suppose

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V})$$

then, by definition of $\mathbf{A}_{\mathrm{C}}$,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V}) = \mathbf{e}^{\mathrm{T}}\mathbf{U}\mathbf{V}^{-\mathrm{T}}$$

and therefore,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}\mathbf{V}^{-\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

or

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

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which is axiom F indeed. All steps can be reversed, proving necessity. This completes the proof.

For the lump-sum (or aggregation) method we have the following result.

Corollary 1
The lump-sum method fulfils the material balance axiom if the total output of industry j matches the total output of commodity j, for all $\mathbf{U}$ and $\mathbf{V}$ with $\mathbf{q}_j = \mathbf{g}_j$, $\forall j$.

Proof
Under the lump-sum method the $\mathbf{A}$-matrix is defined as:

$$\mathbf{A}_{\mathrm{L}}(\mathbf{U}, \mathbf{V}) = \mathbf{U}\hat{\mathbf{g}}^{-1}$$

and, therefore, if we assume that $\mathbf{Ve} = \mathbf{V}^{\mathrm{T}}\mathbf{e}$, that is all $\mathbf{g}_j = \mathbf{q}_j$, we obtain:

$$\mathbf{A}_{\mathrm{L}}(\mathbf{U}, \mathbf{V})\mathbf{q} = \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{q} = \mathbf{U}\hat{\mathbf{q}}^{-1}\mathbf{q} = \mathbf{U}\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{e} = \mathbf{Ue}$$

since $\mathbf{q} = \hat{\mathbf{q}}\mathbf{e}$.
This completes the proof.

We conclude that the material balance axiom will be fulfilled in the lump-sum model if, for all j, total industry output is equal to total commodity output. The reverse does not hold; axiom M does not imply that $\mathbf{g}_j = \mathbf{q}_j$.
The Stone (or by-product) technology model also admits a specific result.

Corollary 2
The Stone (or by-product) technology model fulfils the financial balance axiom for all $\mathbf{U}$ and $\mathbf{V}$ if and only if value-added is zero.

$$\mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

Proof
Under the financial balance axiom, the by-product technology model should verify,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{B}}(\mathbf{U}, \mathbf{V})\,\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

with the left-hand side of this equality such as,

$$\mathbf{e}^{\mathrm{T}}(\mathbf{U} - \tilde{\mathbf{V}}^{\mathrm{T}})\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} = (\mathbf{e}^{\mathrm{T}}\mathbf{U} - \mathbf{e}^{\mathrm{T}}\tilde{\mathbf{V}}^{\mathrm{T}})\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} - \mathbf{e}^{\mathrm{T}}\tilde{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}}$$

Moreover, since $\tilde{\mathbf{V}}^{\mathrm{T}} = \mathbf{V}^{\mathrm{T}} - \hat{\mathbf{V}}^{\mathrm{T}}$ and $\hat{\mathbf{V}}^{\mathrm{T}} = \hat{\mathbf{V}}$ it yields,

$$\mathbf{e}^{\mathrm{T}}\mathbf{U}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} - \mathbf{e}^{\mathrm{T}}\tilde{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} - \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} + \mathbf{e}^{\mathrm{T}}\hat{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}}$$

which is the same as,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{B}}(\mathbf{U}, \mathbf{V})\mathbf{V}^{\mathrm{T}} = (\mathbf{e}^{\mathrm{T}}\mathbf{U} - \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}})\hat{\mathbf{V}}^{-1}\mathbf{V}^{\mathrm{T}} + \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

So, let us assume now that $\mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$, then

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{B}}(\mathbf{U}, \mathbf{V})\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} = \mathbf{e}^{\mathrm{T}}\mathbf{U}$$

All steps can be reversed, proving necessity.

This completes the proof.

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The last model we consider is the mixed technology model presented by ten Raa *et al.* (1984), where the make matrix is split into a table $\mathbf{V}_1$ of primary products and ordinary secondary products, i.e. those products that involve an alternative activity and which are not being generated automatically by the primary productive process, and a table $\mathbf{V}_2$ of by-products.

Kop Jansen & ten Raa (1990) demonstrate that both axioms *M* and *F* hold if and only if the mixed technology model proposed by ten Raa *et al.* (1984) reduces to the commodity technology model. In other words, both axioms hold only when the $\mathbf{V}_2$ table is null, i.e. when there are no by-products, since according to the authors, the so-called ordinary secondary products are included in table $\mathbf{V}_1$.

But what happens in the presence of by-products? Under what restrictions on the data will axioms *M* and *F* still hold? We will take some preliminary results from Kop Jansen & ten Raa (1990) as our point of departure in order to cast light on this issue.

Corollary 3

The CB-Mixed (ten Raa *et al.*, 1984) technology model fulfils the material balance axiom for all $\mathbf{U}$ and non-singular $\mathbf{V}_1$ when $\mathbf{U} = \mathbf{V}^T$.

Proof

Under the CB-Mixed technology model construct, the material balance axiom should verify that,

$$\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e} = (\mathbf{U}-\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T\mathbf{e} = \mathbf{U}\mathbf{e}$$

where $\mathbf{V}_1$ stands for the primary outputs and those secondary products considered as 'ordinary' according to ten Raa *et al.*'s (1984) definition, and $\mathbf{V}_2$, for the by-products. Since we are assuming that $\mathbf{U} = \mathbf{V}^T = \mathbf{V}_1^T + \mathbf{V}_2^T$ it can be shown that,

$$(\mathbf{U}-\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T\mathbf{e} = \mathbf{V}_1^T\mathbf{V}_1^{-T}\mathbf{V}^T\mathbf{e} = \mathbf{U}\mathbf{e}$$

This completes the proof.

Corollary 4

The CB-Mixed (ten Raa *et al.*, 1984) technology model fulfils the financial balance axiom if and only if value-added is zero.

$$\mathbf{e}^T\mathbf{V}^T = \mathbf{e}^T\mathbf{U}$$

Proof

As Kop Jansen & ten Raa (1990) demonstrate, under the CB-Mixed technology model construct the financial balance axiom should verify that,

$$\mathbf{e}^T\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T = \mathbf{e}^T(\mathbf{U}-\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T = \mathbf{e}^T\mathbf{U}$$

This can also be expressed as,

$$\mathbf{e}^T\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T = \mathbf{e}^T(\mathbf{U}-\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T = (\mathbf{e}^T\mathbf{U}-\mathbf{e}^T\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T$$

and substituting $\mathbf{e}^T\mathbf{V}^T = \mathbf{e}^T\mathbf{U}$, it yields,

$$\begin{aligned}\mathbf{e}^T\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T &= (\mathbf{e}^T\mathbf{V}^T-\mathbf{e}^T\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T = \mathbf{e}^T(\mathbf{V}^T-\mathbf{V}_2^T)\mathbf{V}_1^{-T}\mathbf{V}^T\\ &= \mathbf{e}^T\mathbf{V}_1^T\mathbf{V}_1^{-T}\mathbf{V}^T = \mathbf{e}^T\mathbf{V}^T = \mathbf{e}^T\mathbf{U}\end{aligned}$$

All steps can be reversed, proving necessity.

This completes the proof.

Table 4. Additional assumptions over axioms according to models

Model	Axiom *M*	Axiom *F*	Axiom *P*	Axiom *S*
Transfer	$\mathbf{A}_T(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e} = \mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_T(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	Never
By-product technology	$\mathbf{A}_B(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e} = \mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_B(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$ or $\mathbf{e}^T\mathbf{V}^T = \mathbf{e}^T\mathbf{U}$	✓	✓
European System	✓	$\mathbf{e}^T\mathbf{A}_E(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	✓	Never
Lump-Sum	$\mathbf{A}_L(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e} = \mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$ or $\mathbf{V}\mathbf{e} = \mathbf{V}^T\mathbf{e}$	$\mathbf{e}^T\mathbf{A}_L(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	✓
Commodity technology	✓	✓	✓	✓
Industry technology	✓	$\mathbf{e}^T\mathbf{A}_I(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$	Never	Never
ten Raa *et al.* method	$\mathbf{A}_{CB}(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e} = \mathbf{A}_C(\mathbf{U},\mathbf{V})\mathbf{V}^T\mathbf{e}$ or $\mathbf{U} = \mathbf{V}^T$	$\mathbf{e}^T\mathbf{A}_{CB}(\mathbf{U},\mathbf{V}) = \mathbf{e}^T\mathbf{A}_C(\mathbf{U},\mathbf{V})$ or $\mathbf{e}^T\mathbf{V}^T = \mathbf{e}^T\mathbf{U}$	✓	✓

5. Summary and Conclusions

The most interesting conclusion is that the material and financial axioms will be fulfilled under some restrictions on the data (Theorems 1 and 2). For the lump-sum method the material balance is fulfilled if the total commodity outputs match respective total industry outputs. A brief summary of the main results is presented in Table 4. The transfer and the industry technology model need restrictive conditions in order to fulfil all axioms.

Notes

1. The derivation of use and make matrices was first given by van Rijckeghem (1967) and a noteworthy precursor is Edmonston (1952).
2. For a more detailed explanation of the consequences of each one of these methods on the construction of input–output tables, see also Viet (1994).
3. Actually, under the commodity technology model, not only are inputs structures of industries proportional to their outputs, but also the outputs structures of the commodities they produce.

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Appendix

This Appendix proves that the established hybrid constructs on the basis of commodity and industry technology assumptions fulfil only the material balance. As in Kop Jansen & ten Raa (1990) and using their same imaginary use and make matrices, it also presents counterexamples that violate the financial balance, price or scale invariance axioms. Let us define the following use and make matrices to generate counterexamples

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$$\mathbf{U}=\begin{pmatrix}1/2 & 0\\ 1 & 1/2\end{pmatrix},\quad \mathbf{V}=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}\quad \text{and } \mathbf{p}=\mathbf{s}=\begin{pmatrix}2\\ 1\end{pmatrix}$$

In addition, the make table is split into the following $\mathbf{V}_1$ and $\mathbf{V}_2$ matrices,

$$\mathbf{V}_1=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\mathbf{I},\ \mathbf{V}_2=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$

A straightforward computation shows that,

$$\mathbf{g}=\mathbf{Ve}=\begin{pmatrix}2\\ 1\end{pmatrix},\ \mathbf{g}_1=\mathbf{V}_1\mathbf{e}=\begin{pmatrix}1\\ 1\end{pmatrix},\ \mathbf{q}=\mathbf{V}^{\mathrm{T}}\mathbf{e}=\begin{pmatrix}1\\ 2\end{pmatrix},\ \mathbf{q}_2=\mathbf{V}_2^{\mathrm{T}}\mathbf{e}=\begin{pmatrix}0\\ 1\end{pmatrix}$$

and therefore,

$$\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})=\begin{pmatrix}1/4 & 1/8\\ 1/2 & 1/2\end{pmatrix}\quad \text{and}\quad \mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})=\begin{pmatrix}1/4 & 1/8\\ 0 & 3/4\end{pmatrix}$$

with $\mathbf{H}=\begin{pmatrix}2 & 0\\ -1 & 1\end{pmatrix}$.

Material balance (*Axiom* M)

The material balance equation for $\mathbf{A}_{\mathrm{H}}$ is verified as follows:

$$\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}=\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\mathbf{q}=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{I}-\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2)+\mathbf{V}_2\hat{\mathbf{q}}^{-1})\mathbf{q} \tag{4}$$

Since it is true that $\hat{\mathbf{q}}^{-1}\mathbf{q}=\mathbf{e}$, $\mathbf{g}_2=\mathbf{V}_2\mathbf{e}$ and $\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}_2\mathbf{q}=\mathbf{q}_2$, equation (4) can be written as:

$$\begin{aligned}\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}&=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}-\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_2+\mathbf{g}_2)=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{q}-\mathbf{q}_2)+\mathbf{g}_2)\\ &=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1+\mathbf{g}_2)\end{aligned} \tag{5}$$

and if we substitute $\mathbf{q}_1=\mathbf{V}_1^{\mathrm{T}}\mathbf{e}$ in equation (5) bearing in mind that $\mathbf{g}_1=\hat{\mathbf{g}}_1\mathbf{e}$,

$$\begin{aligned}\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}&=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1+\mathbf{g}_2)=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{e}+\mathbf{g}_2)=\mathbf{U}\hat{\mathbf{g}}^{-1}(\mathbf{g}_1+\mathbf{g}_2)\\ &=\mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{g}=\mathbf{Ue}\end{aligned}$$

For $\mathbf{A}_{\mathrm{Y}}$, axiom M is verified as follows:

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}&=\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\mathbf{q}=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{I}-\mathbf{V}_2^{\mathrm{T}}\hat{\mathbf{g}}^{-1}\mathbf{H})+\mathbf{V}_2\hat{\mathbf{q}}^{-1})\mathbf{q}\\ &=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}-\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{V}_2^{\mathrm{T}}\hat{\mathbf{g}}^{-1}\mathbf{Hq}+\mathbf{V}_2\hat{\mathbf{q}}^{-1}\mathbf{q})\end{aligned} \tag{6}$$

Since $\hat{\mathbf{q}}^{-1}\mathbf{q}=\hat{\mathbf{g}}^{-1}\mathbf{g}=\mathbf{e}$, $\mathbf{g}_2=\mathbf{V}_2\mathbf{e}$ and $\mathbf{g}=\mathbf{Hq}$ (see Armstrong, 1975, p. 75) equation (6) can be written as:

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e}&=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}-\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{V}_2^{\mathrm{T}}\mathbf{e}+\mathbf{g}_2)=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}(\mathbf{q}-\mathbf{q}_2)+\mathbf{g}_2)\\ &=\mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1+\mathbf{g}_2)\end{aligned} \tag{7}$$

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for $\mathbf{V}_2^{\mathrm{T}}\mathbf{e} = \mathbf{q}_2$.

If we substitute $\mathbf{q}_1 = \mathbf{V}_1^{\mathrm{T}}\mathbf{e}$ in equation (7), bearing in mind that $\mathbf{g}_1 = \hat{\mathbf{g}}_1\mathbf{e}$,

$$\begin{aligned}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} &= \mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{V}_1^{-\mathrm{T}}\mathbf{q}_1 + \mathbf{g}_2) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\hat{\mathbf{g}}_1\mathbf{e} + \mathbf{g}_2) = \mathbf{U}\hat{\mathbf{g}}^{-1}(\mathbf{g}_1 + \mathbf{g}_2)\\ &= \mathbf{U}\hat{\mathbf{g}}^{-1}\mathbf{g} = \mathbf{U}\mathbf{e}\end{aligned}$$

Financial balance (*Axiom* F)

For both hybrid technology assumptions, the financial balance equation is not fulfilled since,

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}} = (1 \quad 1)\begin{pmatrix}1/4 & 1/8\\ 1/2 & 1/2\end{pmatrix}\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} = (11/8 \quad 5/8),$$

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}} = (1 \quad 1)\begin{pmatrix}1/4 & 1/8\\ 0 & 3/4\end{pmatrix}\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} = (9/8 \quad 7/8)$$

and

$$\mathbf{e}^{\mathrm{T}}\mathbf{U} = (1 \quad 1)\begin{pmatrix}1/2 & 0\\ 1 & 1/2\end{pmatrix} = (3/2 \quad 1/2)$$

Price invariance (*Axiom* P)

The price invariance axiom is violated since,

$$\mathbf{A}_{\mathrm{H}}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \begin{pmatrix}1/3 & 0\\ 1/3 & 1/2\end{pmatrix}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1/2\end{pmatrix} + \begin{pmatrix}0 & 1/2\\ 0 & 0\end{pmatrix}\right] = \begin{pmatrix}1/3 & 1/6\\ 1/3 & 5/12\end{pmatrix},$$

$$\mathbf{A}_{\mathrm{Y}}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \begin{pmatrix}1/3 & 0\\ 1/3 & 1/2\end{pmatrix}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 1/2\\ 0 & 0\end{pmatrix}\right] = \begin{pmatrix}1/3 & -1/6\\ 1/3 & 1/3\end{pmatrix}$$

but

$$\hat{\mathbf{p}}\mathbf{A}_{\mathrm{H}}(\mathbf{U},\mathbf{V})\hat{\mathbf{p}}^{-1} = \begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1/4 & 1/8\\ 1/2 & 1/2\end{pmatrix}\begin{pmatrix}1/2 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1/4 & 1/4\\ 1/4 & 1/2\end{pmatrix}$$

and

$$\hat{\mathbf{p}}\mathbf{A}_{\mathrm{Y}}(\mathbf{U},\mathbf{V})\hat{\mathbf{p}}^{-1} = \begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1/4 & 1/8\\ 0 & 3/4\end{pmatrix}\begin{pmatrix}1/2 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1/4 & 1/4\\ 0 & 3/4\end{pmatrix}$$

Scale invariance (*Axiom* S)

According to the real sphere theorem in Kop Jansen & ten Raa (1990) the material balance and scale invariance axioms characterize the commodity technology model. Hence, if it has been proved that material balance holds under hybrid technology assumptions, necessarily the scale invariance axiom must not hold. Otherwise, commodity technology model must be imposed. Actually, the scale invariance axiom holds when the hybrid technology model is reduced to the commodity technology model.

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