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Homothetic utility, Roy's Lemma and consumer's surplus

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HIGHLIGHTS

- Under homothetic utilities consumer's surplus normalized by income offers an "exact" measure of welfare changes.
- The analysis is at the intermediate level of undergraduate Microeconomics.
- Simultaneous price and income changes are consolidated in a single measure.

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ABSTRACT

I provide a short proof that consumer's surplus normalized by income is the correct welfare measure when utility is homothetic.

1. Introduction

Price effects on consumer's welfare are measured by Hicksian variations or by Marshallian consumer's surplus. Hicksian variations are compensating or equivalent. A compensating variation is an income subsidy that compensates a price increase and an equivalent variation is an income tax which yields an equivalent reduction of well-being. For normal goods an equivalent variation is smaller than the compensating variation, because money is more valuable before the price increase. Because the Hicksian variations have these crisp monetary interpretations, they are called "exact". Marshallian consumer's surplus is the area between the old and new price lines and the ordinary demand curve. It is easy to measure, falls between equivalent and compensating variations, and considered a good "approximation" (Willig, 1976). In a neglected paper Takayama (1982) has shown that not Hicksian variations are the correct theoretical measures, but a variant of consumer's surplus, at least for homothetic demands. This note presents a short alternative proof.

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2. Analysis

A function U is *homothetic* if $U(x) = f(h(x))$, where x is an n-dimensional vector, h a homogeneous function of degree $d > 0$ and f an increasing function. If U is a utility function, we may just as well use any increasing transformation of U. Consequently we may set the degree of the homogeneous function equal to 1 and choose the logarithm for the increasing function. Hence a homothetic utility function U can be written

$$U(x) = \ln(u(x)) : u(sx) = su(x)(s > 0). \tag{1}$$

The essence of a homothetic utility function is that the solution of max $U(x) : px \leq I$ (p a price vector and I income, both positive) is $x = D(p)I$: Demand is multiplicatively separable in price and income. By Eq. (1) indirect utility, which is defined by $V(p, I) = U(x) = U(D(p)I)$, is additively separable:

$$V(p, I) = \ln(u(D(p)I)) = \ln(Iu(D(p))) = V(p, 1) + \ln I. \tag{2}$$

According to *Roy's Lemma* the local price effect on indirect utility equals minus the quantity purchased of that commodity, times the marginal indirect utility of money, which is given by Eq. (2):

$$\frac{\partial V(p, I)}{\partial p} = -x * \frac{\partial V(p, I)}{\partial I} = -D(p)I/I. \quad (3)$$

Here $\frac{\partial V(p,I)}{\partial p}$ is the vector of partial derivatives with respect to the price components and $\frac{\partial V(p,I)}{\partial I}$ the marginal indirect utility of income. In view of Eqs. (2) and (3) the effect of a price change from p^0 to p^1 on indirect utility is given by the line integral

$$V(p^1, 1) - V(p^0, 1) = \int_{p^0}^{p^1} \frac{\partial V(p, 1)}{\partial p} dp = -\int_{p^0}^{p^1} D(p)Idp/I. \quad (4)$$

Measure (4) is the integral of demand $D(p)I$, i.e. *consumer's surplus*, normalized by income (Takayama, 1982). *This* measure is exact, not the Hicksian variations. And since the latter variations are not exact for homothetic utility functions, there is no hope they are exact in general.

A referee insightfully suggested the analysis remains valid when income changes too, from I^0 to I^1. Indeed, in this case Eqs. (2) and (4) yield that indirect utility effect (4) becomes

$$\begin{aligned} V(p^1, I^1) - V(p^0, I^0) &= V(p^1, 1) + \ln I^1 - [V(p^0, 1) + \ln I^0] \\ &= \int_{p^0}^{p^1} \frac{\partial V(p, 1)}{\partial p} dp + \ln(I^1/I^0). \end{aligned} \quad (5)$$

The first term on the right hand side of Eq. (5) is, again, consumer's surplus normalized by income, Eq. (4). It is augmented by the logarithmic growth factor of income.

References

Takayama, A., 1982. On consumer's surplus. Econom. Lett. 10, 35–42.
Willig, R.D., 1976. Consumer's surplus without apology. Am. Econ. Rev. 66 (4), 589–597.